

# Display Advertising Pricing in Exchange Markets

Hana Choi\*

Carl F. Mela<sup>†</sup>

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## Abstract

This paper considers how a publisher should set reserve prices for real-time bidding (RTB) auctions when selling display advertising impressions through ad exchanges, a \$40 billion market and growing. Through a series of field experiments, we find that setting the reserve price increases publisher's revenues by 32%, thereby affirming the importance of reserve price in maximizing publisher's revenues from auctions. Further, we find that advertisers increase their bids in response to an experimental increase in reserve price, and show this behavior is consistent with the use of a minimum impression constraint to ensure advertising reach.

Based on this insight, we construct an advertiser bidding model and use it to infer the overall demand curve for advertising as a function of reserve prices. Using this demand model, we solve the publisher pricing problem. Incorporating the minimum impression constraint into the reserve price setting process yields a 50% increase over a solution that does not incorporate the constraint, and an additional increase in profits of nine percentage points.

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\*Hana Choi is a marketing Ph.D. candidate at the Fuqua School of Business, Duke University (email: hana.choi@duke.edu, phone: 336-392-2288).

<sup>†</sup>Carl F. Mela is the T. Austin Finch Foundation Professor of Business Administration at the Fuqua School of Business, Duke University (email: mela@duke.edu, phone: 919-660-7767).

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The most recent versions of this manuscript will be made available at [hanachoi.github.io](https://github.com/hanachoi)

# 1 Introduction

## 1.1 Overview

Display advertising (display ad herein) markets exist to match advertisers to publishers. This paper considers how a publisher should set reserve prices for auctions when selling display advertising impressions through ad exchanges, a topic of growing economic importance. These markets are estimated to be \$39.4 billion in the US in 2017, growing considerably from \$31.2 billion in 2016.<sup>1</sup> Drivers behind this 26% double-digit growth rate include an upswing in mobile activities, a proliferation in online video ad formats, and technological advancements in real-time buying through ad exchange auctions. Display ad spending is forecasted to surpass even search ad spending and continue its rapid revenue ascent.<sup>2</sup> In spite of this, empirical research into publisher's strategies and advertiser behaviors in display markets is limited. This paper seeks to begin filling this void.

Publishers in display ad markets (e.g., CNN, Wall Street Journal, Facebook, Youtube) are selling a growing number of their advertising impressions on their sites through ad exchanges.<sup>3,4</sup> *Ad exchanges* (e.g., DoubleClick Ad Exchange, OpenX, Rubicon) are centralized marketplaces where publishers supply and advertisers demand display advertising impressions in real time, as consumers browse the pages upon the publishers' sites. To achieve this aim, the publisher first makes the impression available to the ad exchange. Second, the ad exchange then runs an auction for this incoming impression and requests bids from the advertisers. Ad exchanges typically conduct a second-price, sealed-bid auction for each impression instantaneously as it becomes available, thus selling and buying through ad exchanges is often called real-time buying or real-time bidding (RTB). Finally, the winner's ad is served to the consumer. The entire process of selling, buying, and ad serving is typically done within milliseconds.<sup>5</sup>

<sup>1</sup> [https://www.iab.com/wp-content/uploads/2018/05/IAB-2017-Full-Year-Internet-Advertising-Revenue-Report.REV\\_.pdf](https://www.iab.com/wp-content/uploads/2018/05/IAB-2017-Full-Year-Internet-Advertising-Revenue-Report.REV_.pdf)

<sup>2</sup> Search advertising revenue totaled \$40.6 billion in 2017, a little above display advertising, but the growth rate is abating (17%).

<sup>3</sup> <https://www.emarketer.com/content/more-than-80-of-digital-display-ads-will-be-bought-programmatically-in-2018>

<sup>4</sup> In 2018, RTB accounts for 35% (\$14 billion) of the total display ad market in the US, and its share of the display market is quickly increasing.

<sup>5</sup> <http://data.iab.com/ecosystem.html>

One reason RTB is an ascendant approach to selling display advertising is that it offers advertisers a litany of information regarding the characteristics, interests, and past behaviors of the person seeing the impression on the publisher’s page, thereby affording advertisers considerable latitude when targeting customers. The broad array of impression-level characteristics often integrate information from publishers (e.g., the type of articles read), advertisers (such as past purchase), and intermediaries who collect cookie-level information. Moreover, because each impression is sold individually and in real time, the advertiser has substantial flexibility over both how many exposures to buy (reach) and when and how much to spend (budget). Both the advertisers’ valuations for impressions and constraints such as reach or budget can influence the publisher’s pricing decisions.

Accordingly, we consider how advertisers value specific ad impressions, the consequential implications of own and competing valuations for advertiser bidding, and how advertisers’ bidding behaviors can be affected by reach or budget constraints. Subsequently, we examine how the publisher should price (set their reserve prices) in response to this advertiser demand. To address these two questions, we collect a novel dataset on advertiser bidding behaviors from an ad exchange, and run a series of reserve pricing experiments in conjunction with a major publisher. Using this exogenous variation in reserve pricing, we develop a structural model of advertiser bidding behavior in competitive markets, and derive the optimal pricing policy on part of the publisher selling its inventory into the exchange market.

Using a naive assumption of no constraints to set reserve prices in our experiments increased publisher revenues an average of 32% across 12 studies, and enabled us to subsequently test this naive assumption. Initial findings reveal that bids vary with reserves in a manner consistent with advertisers’ use of minimum impression constraints. In the absence of a reserve, 19% of advertisers appear to face this constraint leading them to increase their bids by about 20%. However, as the reserve prices increase, the percentage of advertisers subject to a binding constraint increases to 32% and their bid premium rises to 25%. The use of a naive assumption of no constraints to set reserve prices leads to prices 27% lower than what

the optimal reserves should be in the face of these impression constraints. We project that our model, which accommodates these constraints, would further improve publisher profits by about 9%.

## 1.2 Relevant Research

Much of the display advertising literature focuses on measuring advertising effectiveness and the attendant consequences for advertisers' ad buying and targeting decisions (Barajas et al. 2016, Hoban and Bucklin 2015, Johnson, Lewis, and Nubbemeyer 2016, Johnson, Lewis, and Reiley 2016, Lewis and Rao 2015, Sahni 2015, Sahni and Nair 2016, Tucker 2014, Goldfarb and Tucker 2011). In contrast, our focus is on advertiser valuations and publisher's pricing strategies in exchange markets.

Early research into display markets considered each auction (i.e., each impression) to be a single-shot game (e.g., Johnson 2013, Celis et al. 2014, Sayedi 2017). Second-price, or variant of second-price, auction formats are studied to provide insights on advertisers' strategies and their implications on publishers' revenues.

More recently, research has considered advertisers who are playing repeated auctions for many impressions across lengthy campaigns that can be affected by a number of practical constraints. These include advertisers who (i) face budget constraints for a given campaign in repeated auctions (Balseiro et al. 2015; Balseiro and Gur 2017), (ii) have limited information about their own and/or others' true valuations (Iyer et al. 2014; Cai et al. 2017), (iii) set a number of impressions to attain (Ghosh et al. 2009), or (iv) set pacing options so that the budget is spent smoothly over a specified time period (Lee et al. 2013; Yuan et al. 2013; Xu et al. 2015). In contrast to much of this research focusing on bidding algorithms, our emphasis is empirical/structural and focuses on the publisher's pricing problem.

In recovering advertisers' valuations in this dynamic setting, we adopt the fluid mean-field equilibrium (FMFE) framework developed in Balseiro et al. 2015. Of note, the solution to the advertiser bidding game only depends on the steady-state distribution of rival bids (and not on the current single auction-specific state or the rivals' individual states). This equilibrium

concept provides a computationally tractable way of modeling advertiser bidding behaviors and competition while capturing the dynamic nature of the advertiser decisions across auctions.<sup>6</sup> We extend this framework to accommodate a minimum impression constraint, and suggest estimation and identification strategies to link the FMFE theory to our empirical context.

Finally, in contrast with most of this prior research, we conduct field experiments to provide exogenous variation to test assumptions used in our model and demonstrate the potential of using theoretical insights to improve auction outcomes. On this dimension, our research is related to Ostrovsky and Schwarz 2011, who conducted a large field experiment in the context of search advertising. They find that setting appropriate reserve prices guided by theory leads to substantial increases in seller revenues. We similarly corroborate the importance of setting reserve prices, but in the context of the publisher’s overall revenue in display ad auctions (instead of search). Different from Ostrovsky and Schwarz 2011, we additionally examine the causal effect of reserve prices on advertiser bidding behaviors and use a structural model to back out advertiser primitives.

### 1.3 Organization

This paper is organized as follows. Section 2 first characterizes our data, demonstrating considerable heterogeneity in advertiser bidding behaviors. Next in Section 3, we present the advertiser bidding model and the publisher pricing model. Section 4 discusses the estimation method and the identification argument in inferring advertiser valuations, and Section 5 presents the estimation results. Finally in Section 6, we compute the optimal reserve prices and the revenue gains.

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<sup>6</sup> Backus and Lewis 2016 and Hendricks and Sorensen 2015 study bidders having a unit demand with multiple opportunities to bid in second-price sealed-bid auctions, and apply the framework to eBay’s market. They also similarly use the belief formation and the mean-field equilibrium concept in Krusell and Smith 1998, Weintraub et al. 2008, and Iyer et al. 2014, but for a unit demand without a (reach or budget) constraint.

## 2 Data

In this section, we first describe the publisher and the data source. Second, we show that advertiser bids differ for different types of impressions. This heterogeneity in bids is instrumental in setting reserve prices and also suggests that advertisers consider information about impressions in their bids. Lastly, as a prelude to a bidding model and to demonstrate the economic value of setting reserves, we consider how reserve prices affect publisher’s revenues and advertisers’ bidding behaviors through a series of field experiments. Of note, using theory based reserves led to a 32% increase in profits, providing concrete evidence there is substantial room to enhance pricing outcomes for the publisher.

### 2.1 The Publisher and Data Source

The data for this study are collected from a large, premium publisher, ranked within the U.S. Top 10 by comScore.<sup>7</sup> The publisher has over 30 brands (sites) internationally, among which we focus on the top 20 U.S.-based sites for our analyses. We further focus our attention on display ads and exclude video in our analyses.<sup>8</sup> The data are collected from January 2016 to August 2017.

The publisher’s ad exchange partner provides a report on auction outcomes (related to this publisher’s ad impressions) at the daily level, and the data provided on the various measures are available as daily averages. These data include advertiser ID, Demand Side Platform (DSP herein) ID,<sup>9</sup> day, site where ad was placed, ad type (ad size, ad location on a page, device), number of bids submitted, number of impressions won, bidding amount, payment amount, and click responses. Thus the observational unit (i.e., dimension) is at the advertiser-DSP-day-site-ad type delivered, and the metrics provided for each observational

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<sup>7</sup> <https://www.comscore.com/Insights/Rankings>

<sup>8</sup> We adopt this sampling criterion due to the publisher’s current selling policy. Video ads are mostly sold-out in advance via the direct sales and are rarely available for sale in the ad exchange. Also the online video ads are often sold with the TV ads as a bundle, which hinders our analyses in the absence of the data on TV ads.

<sup>9</sup> Demand Side Platforms (e.g., MediaMath, DataXu, Turn, Rocket Fuel, Adobe Advertising Cloud) help advertisers by facilitating the real-time bidding process. Recognizing that bid optimization is a difficult managerial problem for the advertisers, DSPs specialize in calculating and submitting bids based on the users’ behavioral data and on targeting criteria provided by the advertiser.

unit are number of bids submitted, number of impressions won, bidding amount, payment amount, and number of clicks received. Focusing on the open auctions in the ad exchange, we have 1,466M observations. While number of impressions won, payment amount, and number of click responses are available for all advertisers, number of bids submitted and the bidding amount are only available for a subset of advertisers who opt-in to reveal (share) their data with the publisher.

## 2.2 Summary Statistics

In this section we provide summary statistics of the data to show there exists considerable variation in advertiser bidding/payment and to explain the reason behind using the payment data in our estimation (as opposed to using the bidding data).

Summary statistics of advertiser buying behaviors are presented in Table 1. At the observational unit level (i.e., advertiser-DSP-day-site-ad type delivered), advertisers on average won 70 impressions at \$1.61 CPM rate. The minimum CPM payment is close to zero, because the publisher currently imposes no reserve price in the auctions.

During the sample period, 14,612 advertisers participated in the auctions. Bids are observed from the 82% of the advertisers who opted-in (default setting) to share their bidding information. Opt-in advertisers pay a higher CPM (\$1.89) than the full sample average (\$1.61), but buy a much smaller number of impressions, constituting about 16% of the total revenues.

Hence, there are selection concerns arising from using the bid CPMs for inferring advertiser valuations in estimation. Fortunately, data are available on the CPM paid (i.e, the second highest bid in an auction). Unlike bid CPM, this CPM paid measure is available from all advertisers. Accordingly, we rely on the payment data in our estimation, and only use number of bids submitted and bid CPM data to present evidence of heterogeneity in the bidding behaviors in the following sub-section 2.3.

Table 1: Summary Statistics of exchange

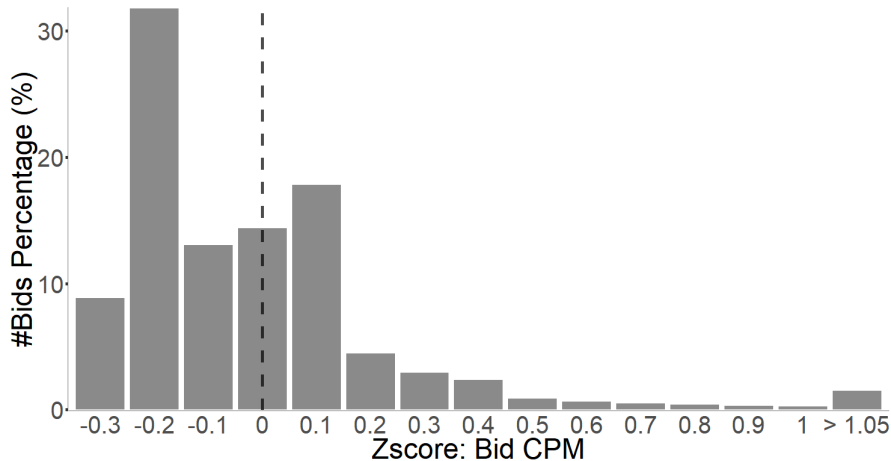
	Per Observation Unit	Mean	Median	Std Dev	Min	Max
All	#Impressions Won	69.9	2	2345	0	587307
	CPM paid (\$)	1.61	0.86	2.74	0.003	191.5
Opt-In	#Impressions Won	10.1	0	1312	0	777817
	CPM paid (\$)	1.89	1.00	3.05	0.002	191.5
	# Bids Submitted	280	5	15042	1	5.4M
	Bid CPM	2.7	1.2	67.4	0.00003	50000

### 2.3 Heterogeneity in Valuations

In order to understand how to set reserve prices, it is imperative for the publisher to determine the distribution of advertiser valuations. The results below suggest substantial heterogeneity in bids with respect to the observed characteristics. To the extent that bids reflect advertisers’ underlying valuations, the optimal reserve prices can potentially be set based on the observed key characteristics driving the heterogeneity for better price discrimination.

First in Figure 1, we explore the distribution of bid CPMs. The x-axis represents zscore of bid CPMs, and the y-axis represents the percentage of number of bids.<sup>10</sup> There exists considerable heterogeneity with the minimum of  $-0.28$  and the maximum value being  $14721.44$ .

Figure 1: Advertiser Bid CPM: Heterogeneity



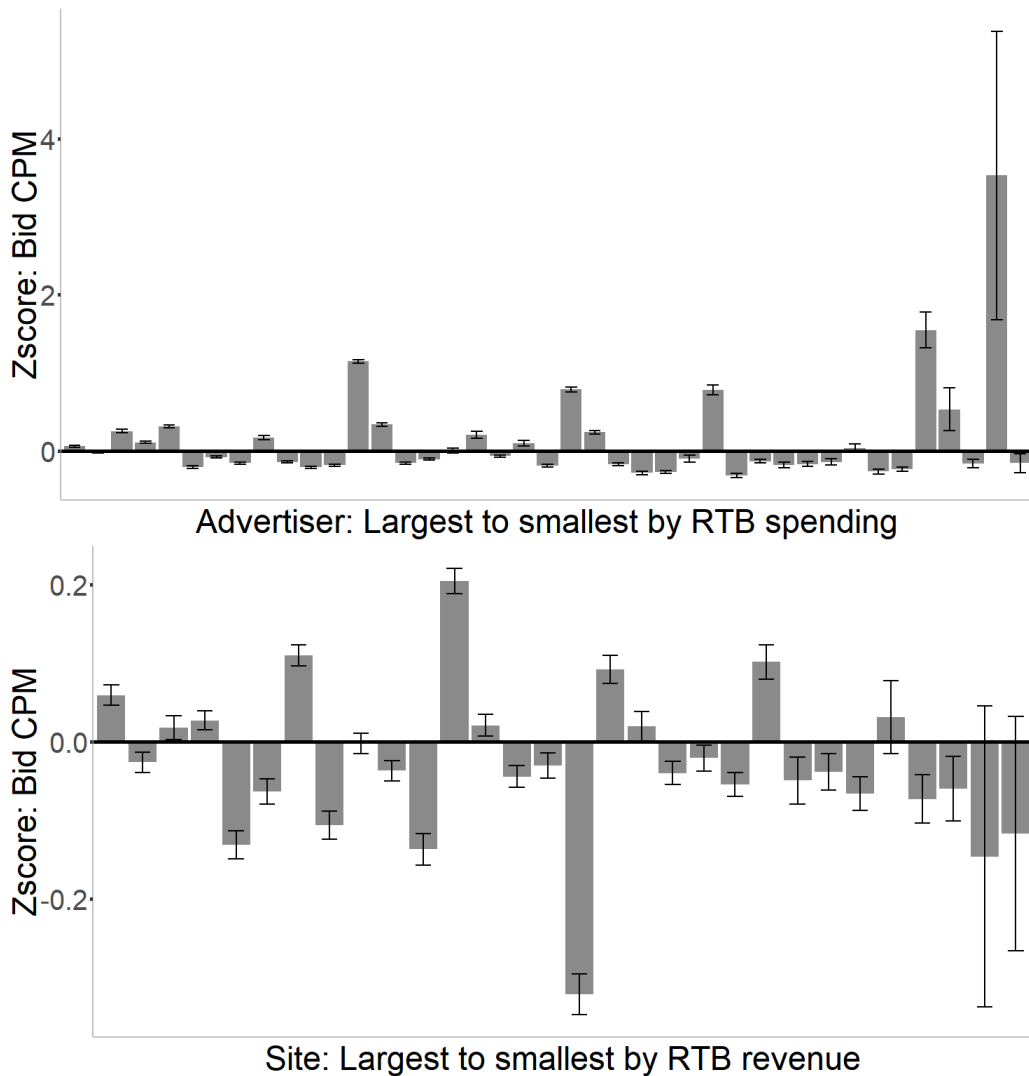
<sup>10</sup> Bid CPM data are daily averages. Thus in calculating the zscore, we use the weighted mean and weighted standard deviation, where the number of bids for a given observational unit is used as the weight.



Table 2: Advertiser Bid CPM: Weighted Least Square Regression

DV: Bid CPM ZScore	Estimates
Desktop	<b>0.059</b> (0.007)
Mobile	<b>-0.062</b> (0.006)
Tablets	<b>-0.057</b> (0.006)
Display vs. App	<b>0.149</b> (0.004)
Above the Fold vs. No info	0.005 (0.004)
Mid vs. No info	<b>-0.012</b> (0.004)
Below the Fold vs. No info	<b>-0.013</b> (0.005)
Site	Yes
Advertiser	Yes
Days (control for supply, competition)	Yes

Figure 2: Advertiser Bid CPM: Advertiser and Site Fixed Effects



To explain the variation in the advertisers’ bid CPMs, we estimate a weighted least square regression of bid CPM zscores on the ad type (ad location, device), site, advertiser, and days. The weight used is the number of bids submitted.<sup>11</sup> Table 2 indicates that advertisers bid more for the desktop ads (relative to mobile, tablets, in-app) and above-the-fold ads (relative to mid or below-the-fold). These results imply that optimal reserve prices can be set differentially based on these ad types. Figure 2 plots the advertiser fixed effects and the site fixed effects estimated from the weighted least square regression of bid CPM zscores. Both advertiser and site explain considerable variation in bid CPMs, suggesting they can also be used to price discriminate via differences in reserve prices.<sup>12</sup>

## 2.4 Evidence for a Minimum Impression Constraint

In setting the optimal reserve prices for the publisher, one needs to understand how advertisers’ bidding behaviors are affected by the reserve prices set by the publisher. Advertisers’ bidding behaviors can be affected by how they value the ad impressions and also by the practical constraints advertisers face regarding reach (number of impressions to buy) or budget. Therefore, a series of field experiments were developed to vary the reserve prices in a set of display auctions, in order to understand their impact on the publisher’s revenues as well as on the advertisers’ bidding behaviors.

The theoretical prediction under the standard second-price, sealed-bid auction format is that bidders bid truthfully, meaning that advertisers’ underlying valuations can directly be inferred from the observed bids. Though the assumption of truth-telling strategy is tractable and often assumed in models of display markets (e.g., Celis et al. 2014, Sayedi 2017), there exists some empirical evidence that advertisers face practical constraints when bidding in the exchange auctions. We begin by outlining the practical constraints advertisers might face when bidding in the exchange. Then we provide evidence that advertisers’ bidding behaviors

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<sup>11</sup> Bid CPM observations are daily averages (i.e., total bidding amount/#bids) for given observational units (advertiser-DSP-day-site-ad type delivered). Thus the WLS regression analysis uses the number of bids for a given observational unit as the weight.

<sup>12</sup> In our empirical analysis, we examine reserve prices based on different ad types and sites, while keeping the reserve price the same across advertisers. This is due to the publisher’s concern for fairness across advertisers and the preference toward a simpler pricing scheme.

are not consistent with truth-telling, but appear to reflect a minimum impression constraint.

### 2.4.1 Practical Constraints in Bidding

Practical constraints discussed in existing literature include (i) budget constraint (Balseiro et al. 2015; Balseiro and Gur 2017), (ii) imperfect information about own and/or other’s true valuations (Iyer et al. 2014; Cai et al. 2017), (iii) minimum impression goal (Ghosh et al. 2009), and (iv) pacing options where the budget is spent smoothly over a specified time period (Lee et al. 2013; Yuan et al. 2013; Xu et al. 2015). Advertisers’ optimal bidding strategies may deviate from their true valuations when faced with these practical constraints. If one or more of these constraints seem to bind advertisers’ bidding behaviors, those constraints will need to be considered in building the advertiser bidding model. Otherwise, predictions of advertisers’ bidding behaviors under the counterfactual (e.g., when increasing the reserve prices) will be incorrect and will lead to wrong inference on the optimal reserve prices.

Table 3: Theory Predictions

Mechanism	Reserve Level		Effect of Imposing Reserve		
	$r = 0$	$r = r_{nc}^* > 0$	Bid CPM	#Impressions Won	Total Payment
Max Budget Constraint	Not Bind	Not Bind	No Change	–	+
	Not Bind	Bind	–	–	+
	Bind	Bind	+/-	+/-	No Change
Min Impression Constraint	Not Bind	Not Bind	No Change	–	+
	Not Bind	Bind	+	–	+/-
	Bind	Bind	+	No Change	+

Our goal then is to change reserve prices and see how advertisers respond in order to determine which mechanism stands out as the main driver behind their behaviors. Table 3 presents the theory predictions on bid CPM, the number of impressions won, and the total payment, when the reserve price is changed from  $r = 0$  (i.e., no reserve) to  $r_{nc}^* > 0$ .  $r_{nc}^*$  is the optimal reserve calculated under the no constraint, single-shot (truth-telling) model.<sup>13</sup> As the reserve price increases from ( $r = 0$ ) to ( $r_{nc}^* > 0$ ), advertisers face tighter constraints, and each row represents a possible scenario of the underlying state: (not bind, not bind), (not bind, bind), (bind, bind). Each mechanism is considered in isolation. That is, when we

<sup>13</sup> Online Appendix B.2 outlines the rationale for these predictions.

consider the budget constraint, we assume that the minimum impression constraint does not bind in both ( $r = 0$ ) and ( $r_{nc}^* > 0$ ).

In sum, by varying the reserve and observing how the metrics change, we can determine which of these explanations is most consistent with the data and ensure our advertiser bidding model specification is consistent with the bidding outcomes.

## 2.4.2 Experimental Setting

Twelve field experiments were conducted on select websites and the pairs (a treatment and a control) were chosen to be closest in terms of ad characteristics, contents, user demographics, revenues, and number of impressions (user visits).<sup>14</sup> The experiments spanned the period 08/01/2017 - 12/03/2017, where 08/01/2017 - 10/15/2017 constitute the ‘Prior’ period and 10/19/2017 - 12/03/2017 constitute the ‘Post’ period in which the reserve prices were changed for the treatment group. The experimental reserve prices in the treatment condition were calculated using a naive theoretical model presuming that advertisers do not face any constraints and that advertisers play a single-shot game for each available impression.<sup>15</sup> During the ‘Post’ period, the reserve prices were set to these calculated levels for the treatment group, while they were kept at the historical levels for the control group (i.e., no reserve prices). The details of the experiment design and calculating the experimental reserve prices can be found in online Appendix A.

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<sup>14</sup> These pairs (a treatment and a control) were randomized either into the treatment or the control group. For example, (site1, desktop, BTF, 300x250) and (site1, desktop, ATF, 300x250) were paired for the first experiment, and the randomly chosen (site1, desktop, BTF, 300x250) was assigned to the treatment group, and (site1, desktop, ATF, 300x250) was assigned to the control group. Details of the experimental groups are included in Table 9 in online Appendix.

<sup>15</sup> Changes in advertiser bidding behaviors with respect to exogenous changes in reserve prices are required to validate theoretical predictions regarding the existence of a budget or impression constraint. However, to validate predictions about how bids can change with respect to constraints, the level of the treatment reserve prices need not be at the optimal level, merely that the reserves vary exogenously. As such, the suggested experimental reserve prices provide the first order approximations of the optimal reserve prices, and are likely to increase the publisher’s revenues close to the global maximum. Further improvements are possible when constrained bidding is considered, as we shall discuss later.

### 2.4.3 Experimental Results on eCPM

We begin by reporting experimental results regarding the publishers’ revenues to gauge the effectiveness of setting reserve prices in auctions.

**Effect on eCPM** The outcome measure considered is *eCPM* (effective CPM, industry vernacular), which yields a per supplied impression revenue.

$$eCPM = \frac{\text{Revenue}}{\# \text{ Impression Supplied to Exchange (in thousand)}}$$

Table 4 shows the treatment effect on eCPM, where eCPM is multiplied by a common, multiplicative constant for confidentiality. The increase in revenues (holding the impressions supplied to exchange the same) is huge, 32%, thereby affirming the importance of setting the reserve prices in running auctions.

$$\begin{aligned} \text{Increase in revenue} &= \frac{(0.49 - 0.38) - (0.33 - 0.34)}{0.38} \\ &= \frac{0.12}{0.38} \simeq 32\% \end{aligned}$$

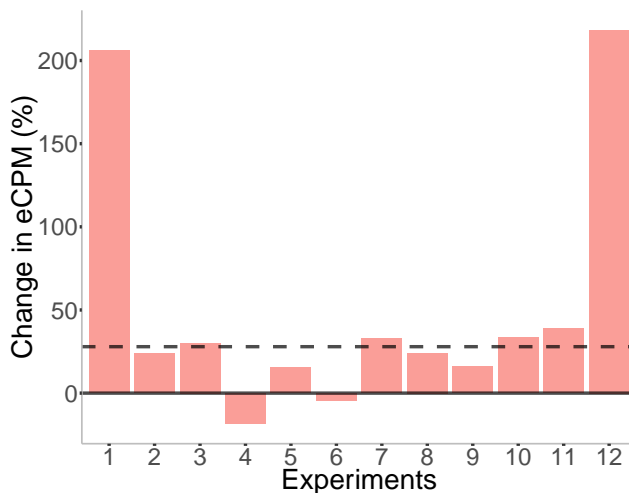
Table 4: Treatment Effect on eCPM

Group	Reserve	Prior (08/01/17 - 10/15/17)	Post (10/19/17 - 12/03/17)
Treatment	Yes	0.38	0.49
Control	No	0.34	0.33

Note: eCPM is scaled by multiplicative constant for confidentiality.

Figure 3 plots the treatment effect by the experimental groups, and includes a dotted horizontal line representing the average percentage change in eCPM across treatments, 32%. Differences in treatment effects across experiments can largely be explained by the observed characteristics of the site-ad type used in the various experiments, and the results from a formal difference-in-difference analysis on eCPM is reported in Table 10 in online Appendix.

Figure 3: Treatment Effect on eCPM by Group



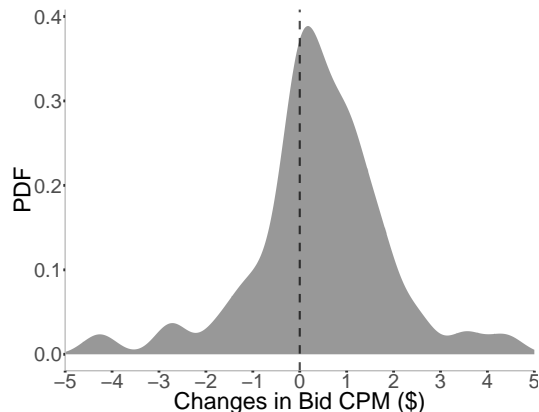
#### 2.4.4 Experimental Results on Bidding Behaviors

To assess how advertisers’ bidding behaviors are affected by the (exogenous) increase in reserve prices, we consider their bid CPM data. If advertisers bid truthfully, the distribution of bids will remain invariant to the experimental manipulation of the reserve price. However, this is not the case. Figure 4 plots the distribution of changes in bids for those advertisers bidding both in the ‘Prior’ and ‘Post’ periods for a given (site-ad type). The changes are computed as the advertiser’s average bid CPM in the ‘Post’ minus that in the ‘Prior’ period. The probability density function’s mass is greater in the positive region, meaning advertisers increase their bids in the ‘Post’ period in response to an increase in the reserve price.<sup>16</sup>

Turning to a difference-in-difference (DiD) analysis of the experimental outcomes for bids, the first column in Table 5 presents the results of a DiD analysis that controls for a time trend before and after the experiments (i.e., day fixed effects to control for the time trend such as seasonality), experiment fixed effects, the interaction of treated and experiment fixed effects (to control for the potential difference between the treatment and control group in the ‘Prior’ period for each experiment), advertiser fixed effects (to control for unobservable

<sup>16</sup> If the advertiser’s cost to participate in the auction is high, the advertiser might not submit bids in the ‘Post’ period should the reserve price exceed its valuation. To account for this truncation in the observed data for the treatment group, we analyze both the prior- and post- bids that are above the reserve price used in the post period, and find a similar pattern.

Figure 4: Treatment Effect on Bid CPM Distribution



advertiser related factors), and other observables.<sup>17,18</sup> The units of observation for the DiD regression are at (advertiser-DSP-day-site-ad type) and are weighted by the number of bids submitted in running the weighted least square regression.<sup>19</sup> Results of the DiD analysis indicate that bid distributions change with respect to the (exogenous) increase in reserve prices, by \$0.06, which suggests that advertisers do not bid truthfully.<sup>20</sup>

Table 5: DiD Regression Results

DV	Bid CPM	# Impressions Won	Total Payment
Treated $\times$ Post	0.06** (0.01)	-0.50 (0.89)	0.007** 0.003
Treated $\times$ Experiment	Y	Y	Y
Day fixed effect	Y	Y	Y
Experiment fixed effect	Y	Y	Y
Site-Adtype controls	Y	Y	Y
Advertiser fixed effect (Top 1000)	Y	Y	Y
DSP fixed effect	Y	Y	Y
Adjusted $R^2$	0.47	0.35	0.33
$N$	4, 712, 375	7, 054, 582	7, 054, 582

Note: \* $p < 0.1$  \*\* $p < 0.05$

<sup>17</sup> From the experiments, we have 4, 712, 375 bid-metrics at the advertiser-DSP-day-site-ad type level from the opt-in advertisers.

<sup>18</sup> Fixed effects are included for the top 1000 advertisers, who constitute 97% of the total revenue. The post-experimental period indicator is not estimated since the day fixed effect is added.

<sup>19</sup> Recall that a bid CPM observation is a daily average (i.e., total bidding amount/#bids) for a given observational unit (advertiser-DSP-day-site-ad type delivered). Thus the DiD regression uses the number of bids for a given observational unit as the weight.

<sup>20</sup> Results are robust to potential selection considerations arising, for example, from not observing bids from opt-out advertisers. Please see online Appendix A.2.2 for this and other DiD analysis robustness checks.

### 2.4.5 Theoretical Rationale for Experimental Results

In addition to the bid CPM analysis reported in Table 5, the DiD analyses are also conducted on the number of impressions won and the total payment. The second and third columns in Table 5 indicate that the number of impressions won stay the same while the total payment increases. Combining these results with the observation that bid CPM increases upon raising the reserve, the findings are suggestive that advertisers do not bid their valuations, but rather set a minimum number of impressions to attain when bidding in the exchange auctions (Table 3). The existence of such a constraint is consistent with the common industry practice of setting and monitoring the ‘win rate’ (i.e., the number of impressions won / the number of bids submitted) as one of the KPIs and our discussions with industry participants.<sup>21</sup>

Based on these findings from the experiments, the advertiser bidding model is constructed to incorporate the minimum impression constraint and departs from the commonly adopted truth-telling strategy.

## 3 Model

With an eye towards linking reserve pricing to publisher revenues in the exchange channel, this section develops a model of advertiser ad buying as a function of advertisers’ valuations and reserve prices. Our approach builds upon the theoretical work of Balseiro et al. 2015, who use a fluid mean-field equilibrium (FMFE)<sup>22</sup> to solve the dynamic optimal bidding problem of budget-constrained advertisers buying multiple impressions for a campaign that spans multiple second-price auctions. In light of our finding that bid CPMs are positively correlated with the reserve prices, we extend this work to consider a FMFE where advertisers are assumed to face a minimum impression constraint. Analogous to a budget constraint, a minimum impression constraint can induce advertiser interactions that influence how reserve prices affect publisher revenues. As we shall show, this constraint induces advertisers to bid higher than their true valuations (subject to a participation constraint).

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<sup>21</sup> <https://www.adtaxi.com/2018/04/24/exchange-win-rates-bigger-always-better/>

<sup>22</sup> See Section 3.2.5 for a discussion of the FMFE.



In what follows, we describe the publisher-advertiser game and then solve it. Using backward induction, we first solve the advertiser’s ad buying problem, and then characterize the publisher’s reserve pricing problem.

### 3.1 Model Overview

The advertising game proceeds as follows:

1. Publisher: The publisher is the leader and selects the reserve price in the second-price auction  $r$ , for the unsold inventory from the direct channel. The publisher’s objective is to maximize the long-run expected revenues from the auctions. The publisher knows the distribution from which advertisers’ valuations are drawn, but does not know the realized true value for each particular impression.
2. Advertisers: The advertisers are the followers and decides how much to bid  $b$ , based on the realized valuation  $v$  for each given impression and the reserve price  $r$ . The advertisers’ objective is to maximize expected utility (= valuation – cost) from the auctions given the minimum impression level,  $y$ .

### 3.2 Assumptions

This sub-section outlines the model assumptions used to solve the game.

#### 3.2.1 Auction Rule

Reflecting industry practice, impression auctions are second-price, sealed-bid and this auction format is set exogenously by the ad exchange. An impression is delivered to the winner who bids the highest above the reserve price (i.e., the winner’s ad is displayed to the consumer). If no advertiser bids above the reserve price, the impression remains unsold and is perishable. The winner pays the second highest bid, or the reserve price if the second highest bid falls below the reserve price.

#### 3.2.2 Independent Private Value (IPV)

Following the independent private value assumption adopted in prior work (e.g., Edelman et al. 2007, Ostrovsky and Schwarz 2011, Balseiro et al. 2015), advertiser valuations are

assumed to be drawn independently from the conditional distribution  $F_{V|Z}(v|z)$ , where  $Z$  is observed characteristics. After controlling for these observed covariates, advertiser valuations are assumed to be independent.<sup>23,24</sup>

### 3.2.3 Reserve Price

The reserve price is assumed to be known to all potential bidders (advertisers). In practice, the reserve price is not announced to the advertisers, but they can infer it from their repeated auction experience (for example, via machine learning algorithms).<sup>25</sup>

### 3.2.4 Utility Function

Advertisers are assumed to have a quasi-linear utility function, where utility is defined as the sum of the advertiser’s valuations from the impressions won less the total payment for the impressions.

### 3.2.5 Fluid Mean-Field Equilibrium (FMFE)

The equilibrium concept we adopt is the fluid mean-field equilibrium (FMFE) as defined in Balseiro et al. 2015. This equilibrium considers a mean-field approximation (Weintraub et al. 2008, Iyer et al. 2014) to relax the informational requirements of agents. This FMFE equilibrium concept considers a stochastic fluid approximation from the revenue management literature that is suitable when the number of bidding opportunities is large (Gallego and Van Ryzin 1994). In a setting where a large number of advertisers compete, the rational expectation of competitors’ bids can be formulated (approximated) based on the aggregate and stationary distribution of others’ bids instead of tracking each individual competitor’s bid.

The FMFE has two benefits. First, it lightens the assumed burden on part of the decision

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<sup>23</sup> This specification allows for values that are common (affiliated) via the covariate  $Z$ . For example,  $Z$  may contain site dummies to capture that site1 is valued more than site2 among the advertisers.  $Z$  can also contain advertiser dummies to control for the heterogeneity across advertisers.

<sup>24</sup> Athey and Haile 2002 note that the conditional independence assumption (i.e., whether there is unobserved auction-specific heterogeneity after controlling for  $Z$ ) can be tested if more than one bid is observed in each auction or the transaction price is observed in auctions with exogenously varying numbers of bidders (Theorem 3). Using the payment data alone, unfortunately, this conditional independence assumption cannot be tested.

<sup>25</sup> <https://www.aarki.com/blog/understanding-hard-and-soft-price-floors-in-programmatic-media-buying>

maker, who needs only to know the distribution of others' bids (as opposed to all others' bids). In large markets, tracking all others' bids would be challenging, if not impossible, for bidders. Second, this approach reduces the computational burden of solving for a market equilibrium.<sup>26</sup>

To employ the FMFE concept, therefore, several assumptions are required. First, the distribution of competitors' bids is assumed to be stationary, conditional on the observed characteristics  $Z$  (which includes time dummies) and the reserve price  $r$ . Second, the bids of a single advertiser do not affect this distribution. Given that a large number of advertisers compete in the exchange channel, and the marginal impact of any player is small, neither of these assumptions appear restrictive.<sup>27</sup>

Third, bidders' minimum impression constraints need only be satisfied in expectation when bidders solve for their optimal bidding strategies.<sup>28</sup> With this third assumption, it can be shown that bidding strategies that do not condition on the individual state of other advertisers' valuations (instead depending only on the bidder's own valuation, the *distribution* of others' valuations, and the reserve) closely approximate the solution to the optimal bidding strategy that specifically conditions on the individual states of others. This third assumption is motivated by the fact that an advertiser has a large number of bidding opportunities over the campaign length.

### 3.3 Advertiser Bidding Model

The goal of the advertiser bidding model is to determine the optimal bidding policy faced by advertisers (which we can match to the data in order to back out the distribution of advertiser valuations in order to explore the effect of reserves).

The amount of advertising inventory varies over time. Following Balseiro et al. 2015, we assume that the arrival process of available advertising impressions ( $\simeq$  users) at any given

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<sup>26</sup> Readers are referred to Balseiro et al. 2015 for more detail and a formal definition of the FMFE. The authors also provide a theoretical justification for using the FMFE as an approximation of advertisers' behaviors in display markets.

<sup>27</sup> This assumption doesn't require that the average number of bidders *per auction* to be large.

<sup>28</sup> That is, when solving the bidding strategy to maximize its expected utility, the advertiser chooses a bidding function that satisfies the minimum impression constraint *ex ante*, in expectation.

point in time follows a Poisson distribution with intensity  $\eta$ . As suggested in sub-section 2.4.5, we allow for advertiser  $k$  to have a minimum impression level  $y_k$  for the campaign length  $s_k$  (i.e., the duration over which an advertiser is using its bidding rule). Then,  $\eta s_k$  (the expected arrivals times the duration of the bidding interval) indicates the total number of impressions arriving during the campaign period.

For a given impression  $i$  that arrives in this bidding interval (campaign length), we denote advertiser  $k$ 's value as  $v_{ik}$ , which is assumed to be drawn independently and identically from a continuous cumulative distribution  $F_V(\cdot|Z)$ .  $Z$  are observed characteristics (e.g., ad type). Further, let  $D$  be the steady-state maximum of the competitors' bids, where the publisher is also considered as one competitor that submits a bid equal to  $r$ . The distribution of  $D$  will endogenously be determined in equilibrium, which we denote as  $F_D$ .

The advertiser maximizes its expected utility (= valuation – cost) from the ad auctions, given the minimum impression constraint and the participation constraint. With the assumptions made in sub-section 3.2, we focus on the bidding strategy  $\beta_\theta^F(v_i|F_D, \theta)$  for an advertiser type  $\theta = (s, y)$  (where advertiser type is defined by the campaign length and the minimum impression level) to be a function of the advertiser's own valuation  $v_i$ . The advertiser faces the optimization problem given by

$$\begin{aligned}
 J_\theta^F(F_D) &= \max_b \eta s_\theta E_{V,D} [\mathbf{1} \{b(V) \geq D\} (V - D)] & (1) \\
 \text{s.t. } y_\theta &\leq \eta s_\theta E_{V,D} [\mathbf{1} \{b(V) \geq D\}] \\
 0 &\leq \eta s_\theta E_{V,D} [\mathbf{1} \{b(V) \geq D\} (V - D)]
 \end{aligned}$$

where the expectation is taken over both  $F_V$  (the distribution of valuations) and  $F_D$  (the distribution of the maximum competing bid). To define a well-behaved optimization problem, we assume  $y_k < \eta s_k$ , that is the minimum impression level is lower than the total available impressions.

In the first line,  $\eta s_\theta$  indicates the total number of impressions arriving during the campaign period.  $\mathbf{1} \{b(V) \geq D\}$  indicates the probability of winning the auction on a given arrival, where the advertiser's bid is higher than the maximum of the competitors' bids. Lastly,

$(V - D)$  indicates valuation minus payment, where the payment is consistent with the second-price rule.

In the second line, the right hand side is the expected number of impressions won at the end of the campaign period. The inequality constraint assures that the expected number of impressions won is greater than the minimum impression level  $y_\theta$ . This inequality can also be written as  $\frac{y_\theta}{\eta s_\theta} \leq E_{V,D} [\mathbf{1} \{b(V) \geq D\}]$ , implying that the advertiser tries to attain a minimum auction winning rate of  $\frac{y_\theta}{\eta s_\theta}$ .

The third line captures the advertiser's participation constraint that its expected utility in bidding in the exchange channel is greater than zero. Below we characterize advertisers' optimal bidding strategies, assuming that the participation constraints hold (do not bind) in equilibrium. In sub-section 3.4, we discuss how the participation constraint is imposed in calculating the optimal reserve price.<sup>29</sup>

**Proposition 1.** *Suppose that  $E[D] < \infty$ . An optimal bidding strategy that solves (1) is given by*

$$\beta_\theta^F(v|F_D) = v + \mu^*$$

where  $\mu^*$  is the optimal solution of the dual problem

$$\inf_{\mu \geq 0} \eta s_\theta E_{V,D} [\mathbf{1} \{V \geq D - \mu\} (V - D + \mu)] - \mu y_\theta$$

That is the advertiser bids higher than its own valuation by a constant factor  $\mu^*$ , which is the optimal dual (Lagrangian) multiplier of the minimum impression constraint. Intuitively, this means that the advertiser foregoes some utility to satisfy the constraint. This constant factor  $\mu^*$  guarantees that the advertiser meets the minimum impression constraint at the end of the campaign period. Of note, the dynamic nature of the repeated auctions is captured by

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<sup>29</sup> In estimation, where advertiser valuations are inferred conditional on observing the data, this assumption implies that the participation constraints do not bind for those observed advertisers at their observed (optimal) bidding strategies given  $r = 0$ . This assumption that the participation constraint does not bind in estimation is not restrictive, as it implies that advertisers who participated in the exchange auctions expected to gain positive utilities at  $r = 0$ . In the counterfactual, where we calculate the optimal reserve price, advertisers might face binding participation constraints as the reserve price increases, thereby potentially dropping out of the auction.

this constant factor  $\mu^*$ , and the bidding strategy becomes static in the sense that  $\mu^*$  does not depend on the current single auction-specific state.<sup>30</sup>

**Proposition 2.** *If the participation constraints do not bind in equilibrium, the equilibrium can be characterized as follows:*

$$\beta_{\theta}^F(v|F_D) = v + \mu^*$$

where  $\mu^*$  is

$$\begin{cases} \mu^* = 0 & \text{if } y_{\theta} < \eta s_{\theta} E_{V,D} [\mathbf{1}\{V \geq D\}] \\ y_{\theta} - \eta s_{\theta} E_{V,D} [\mathbf{1}\{V + \mu^* \geq D\}] = 0 & \text{if } y_{\theta} \geq \eta s_{\theta} E_{V,D} [\mathbf{1}\{V \geq D\}] \end{cases}$$

The proposition states that if the minimum impression constraint is not binding, then in equilibrium advertisers will bid truthfully ( $\mu^* = 0$ ). On the other hand, if the minimum impression constraint does bind, then advertisers will bid higher than the true valuations, where  $\mu^*$  solves the implicit function  $y_{\theta} - \eta s_{\theta} E_{V,D} [\mathbf{1}\{V + \mu^* \geq D\}] = 0$ . Of note,  $y_{\theta} - \eta s_{\theta} E_{V,D} [\mathbf{1}\{V + \mu^* \geq D\}]$  is the number of expected impressions shy of the minimum impression level at the end of the campaign, when the optimal bid function is employed.

Based on this proposition, the cost of the minimum impression constraint,  $\mu^*$ , increases with the minimum impression level ( $y_{\theta}$ ), and decreases with the number of impressions and length of campaign ( $\eta s_{\theta}$ ). Perhaps more importantly for our purposes, an increase in the second highest payment,  $D$ , lowers  $E_{V,D} [\mathbf{1}\{V + \mu^* \geq D\}]$  and thus increases  $\mu^*$ , the bid premium. Because an increase in reserves can increase  $D$  when the reserve binds (it binds when the second highest valuation is lower than the reserve), reserves can lead to higher bids, consistent with our experimental data. The proofs for Proposition 1 and Proposition 2 are in online Appendix B.1.

The primitives to be estimated are  $(F_V, F_D, \boldsymbol{\mu})$  given the reserve price  $r$ .

<sup>30</sup> In the budget constraint case (Balseiro et al. 2015), advertisers will shade down their bids by a constant factor to account for the future bidding opportunities (future value). In our minimum impression constraint case, advertisers shade up by this constant factor  $\mu^*$  to satisfy the constraint at the end of the campaign period.

### 3.4 Optimal Publisher Ad Auction Reserve Price

The publisher's objective is to maximize the long-run expected revenues from the auctions given advertiser valuations. Within the independent private value (IPV) paradigm, the publisher can maximize the revenues from the RTB auctions by choosing the reserve price optimally.

#### 3.4.1 Publisher's Optimization Problem

We denote  $G_\theta(\boldsymbol{\mu}, r) = E_{V,D}[\mathbf{1}\{V + \mu_\theta \geq D\} D]$  to be the expected payment of a  $\theta$ -type advertiser when advertisers bid according to the profile  $\boldsymbol{\mu}$ , and the publisher sets a reserve price  $r$  ( $G_\theta$  term is the product of the payment times the likelihood the impression is won). We define  $I(\boldsymbol{\mu}, r) = F_D(r|\boldsymbol{\mu})$  as the probability that the impression is not won in the exchange.

The publisher's problem can then be written as

$$\begin{aligned} \max_{r,\eta} & \left[ \sum_{\theta} \{p_\theta s_\theta G_\theta(\boldsymbol{\mu}, r)\} + cI(\boldsymbol{\mu}, r) \right] \\ \text{s.t. } & \mu_\theta \geq 0 \perp y_\theta \leq \eta s_\theta E_{V,D}[\mathbf{1}\{V + \mu_\theta \geq D\}] \quad \forall \theta \in \Theta \end{aligned} \quad (2)$$

where  $p_\theta$  is the probability that an arriving advertiser is of type  $\theta$  and  $c > 0$  is the publisher's valuation (i.e., the outside option value if the impression is not won by some advertiser in the exchange).<sup>31</sup>

The first term in the sum in the first line indicates the average expenditure of the advertisers (i.e., the second highest bids when the auctions are won by some advertisers). The second term in the first line indicates the publisher's outside option value when the impression is not won by any advertiser, as it reflects the product of the scrap value of the impression and the probability the impression is not sold. The constraints in the second line reflect the conditions for the Lagrangian multipliers in ensuring the minimum impression constraints (either the constraint binds or it does not).

<sup>31</sup> Note that, by definition,  $I(\boldsymbol{\mu}, r) = F_D(r|\boldsymbol{\mu}) = 1 - \sum_{\theta} \{p_\theta s_\theta E_{V,D}[\mathbf{1}\{V + \mu_\theta \geq D\}]\}$ . Thus, Equation (2) can be interpreted as the publisher's value over selling and not selling an impression.

### 3.4.2 Advertiser Participation Constraints

In the counterfactual, where we increase the reserve price to the optimal level, some advertisers will start to face binding participation constraints as the reserve price increases.<sup>32</sup> To incorporate the effect of the participation constraints in the policy simulation, we ascertain whether the bidding profile  $\boldsymbol{\mu}$  satisfies the participation constraint ( $0 \leq \eta s_\theta E_{V,D} [\mathbf{1}\{V + \mu_\theta \geq D\} (V - D)]$ ) at the considered reserve price level  $r$  for  $\forall \theta \in \Theta$ .

In the case the participation constraint does not hold for some advertiser types  $\theta$ , we calculate the maximum  $\mu_\theta$  that satisfies the participation constraint such that

$$\bar{\mu}_\theta = \max_{\mu_\theta \geq 0} [0 \leq \eta s_\theta E_{V,D} [\mathbf{1}\{V + \mu_\theta \geq D\} (V - D)]]$$

This is, the advertiser increases its bid to  $v + \bar{\mu}_\theta$  to bid as closely as possible to its minimum impression level while satisfying the participation constraint.<sup>33</sup>

Given the publisher's outside option value  $c$ , the primitives to be recovered from the policy simulation are the optimal reserve price  $r^*$  and the corresponding outcomes ( $F_V$ ,  $F_D$ ,  $\boldsymbol{\mu}$ ) at  $r^*$ .

## 4 Identification and Estimation

This sub-section discusses the identification and estimation strategies for the inference of advertiser valuations.

### 4.1 Identification

As the publisher currently imposes no reserve price, the identification strategy is described conditioned on no reserve price.<sup>34</sup> First we characterize the identification strategy when the impression constraint does not bind ( $\mu^* = 0$ ), then we discuss the case of the binding

<sup>32</sup> In the extreme case where all advertisers face binding impressions constraints, advertisers' bidding strategies will reach  $\infty$  as the reserve price increases to  $\infty$  without the participation constraints.

<sup>33</sup> Because of the participation constraints, the advertiser may not achieve the minimum impression level under the counterfactual. In this case, we are assuming that advertisers purchase as many impressions as possible toward the minimum impression level while satisfying the participation constraints. Alternatively, we can specify the advertisers to drop-out all together from the exchange channel when the participation constraints do not meet, but we think the former is more realistic in our context.

<sup>34</sup> When the reserve prices exit, the identification strategy will be similar, but the distribution of bids will represent a truncated distribution of valuations due to endogenous participation.



constraint ( $\mu^* > 0$ ).

#### 4.1.1 Case1: $\mu^* = 0$

The equilibrium-bid function in Proposition 1 implies that advertisers bid truthfully when the minimum impression constraint does not bind. Thus, the probability density function of observed bids can directly be mapped to that of valuations. In other words, the distribution of bids identifies the distribution of valuations such that

$$\begin{aligned} f_V^0(v) &= f_B^0(b) \\ F_V^0(v) &= F_B^0(b) \end{aligned}$$

where  $f_V^0$  and  $F_V^0$  ( $f_B^0$  and  $F_B^0$ ) represent probability density function and cumulative density function of valuations (bids) respectively. The superscripts “0” denote the true population values.

Because only the payment data are used in estimation (see sub-section 2.2), the distribution of the observed payments needs to be linked to the distribution of valuations. Denoting  $D$  to be the winning payments,  $F_D^0$  is defined as the distribution of the second highest bids. The distribution of order statistics implies that

$$\begin{aligned} F_D^0(v) &= n(n-1) \int_0^{F_V^0(v)} u^{n-2}(1-u) du \\ &\equiv \varphi(F_V^0(v)|n) \\ f_D^0(v) &= n f_v^0(v)(n-1) F_V^0(v)^{n-2} (1 - F_V^0(v)) \end{aligned}$$

In the last line,  $n f_v^0(v)$  indicates that one of the  $n$  advertisers (i.e.,  $\binom{n}{1} = n$ ) draws  $v$  exactly, and  $(n-1) F_V^0(v)^{n-2} (1 - F_V^0(v))$  indicates that  $(n-2)$  out of the remaining  $(n-1)$  advertisers (i.e.,  $\binom{n-1}{n-2} = n-1$ ) draw valuations lower than  $v$ ,  $(F_V^0(v)^{n-2})$ , and 1 advertiser draws valuation higher than  $v$ ,  $(1 - F_V^0(v))$ .

Since  $\varphi(\cdot|n)$  is a strictly monotonic function given  $n$ ,  $F_V^0$  is identified from the distribution of observed winning payments  $F_D^0$  when the number of potential bidders  $n$  is known (Paarsch,

Hong, et al. 2006).

#### 4.1.2 Case2: $\mu^* > 0$

When the minimum impression constraint binds, the distribution of bids identifies the distribution of valuations up to a location constant  $\mu^*$  such that

$$\begin{aligned} f_W^0(w) &= f_B^0(b) \\ F_W^0(w) &= F_B^0(b) \end{aligned}$$

where  $w = v + \mu^*$  and  $\mu^*$  are the optimal Lagrangian multipliers. Thus  $f_V(v) = f_W(v + \mu^*)$ . Assuming we observe the advertiser type  $\theta = (s, y)$ ,  $\mu^*$  can be estimated from the conditions in Proposition 2, such that  $\mu^*$  solves  $y_\theta - \eta s_\theta E_{V,D}[\mathbf{1}\{V + \mu_\theta^* \geq D\}] = 0$  if  $y_\theta \geq \eta s_\theta E_{V,D}[\mathbf{1}\{V \geq D\}]$ .

Combining the argument in Case 1,  $F_V^0$  is identified from the distribution of observed winning payments  $F_D^0$  when the number of potential bidders  $n$  and the advertiser types (i.e., minimum impression level, campaign duration) are known.

## 4.2 Estimation

The estimation strategy is detailed in this subsection. As in the preceding section, we discuss the case when the constraint does not bind ( $\mu^* = 0$ ) first, then incorporate the case of the binding constraint ( $\mu^* > 0$ ).

#### 4.2.1 Case1: $\mu^* = 0$

Under the truth-telling scenario,  $F_D^0$  can be estimated by substituting the sample analogue for the population quantity as

$$\begin{aligned} \hat{F}_D(v) &= \frac{1}{T} \sum_{t=1}^T \mathbf{1}[d_t \leq v] \\ &= n(n-1) \int_0^{\hat{F}_V(v)} u^{n-2}(1-u) du \end{aligned} \tag{3}$$

The asymptotic properties of the estimator  $\hat{F}_D(v)$  (and the resulting  $\hat{F}_V$ ) are discussed in Paarsch, Hong, et al. 2006.

Incorporating (discrete) covariates is possible as follows. For a given characteristic  $z \in \mathbf{Z}$ ,

the estimator of  $F_{V|Z}^0(v|z)$  is specified as

$$\hat{F}_{D|Z}(v|z) = n(n-1) \int_0^{\hat{F}_{V|Z}(v|z)} u^{n-2}(1-u) du \quad (4)$$

Because the number of participants vary across auctions,  $n$ -specific non-parametric empirical cumulative distribution functions (ECDFs) are estimated first for a particular combination of the  $z$ . Then  $\hat{F}_{V|Z}(v|z)$  is obtained by kernel smoothing  $n$ -specific ECDFs. The exceptionally large number of payments observed given a particular combination of ad characteristics facilitates inference in this context of display ad markets.<sup>35</sup>

#### 4.2.2 Case2: $\mu^* > 0$

The key to incorporating the minimum impression constraint is estimating  $F_{V|Z}$  together with  $\mu^*$  for a given  $z$ . To simplify notation, we drop the dependence on  $z$ . The estimation is done in two stages. In the first stage, we estimate location shifted distribution  $F_V^0(w)$  where  $w = v + \mu$ . In the second stage, conditioned on the recovered distribution  $F_V^0(w)$ ,  $\mu$  is estimated using the condition in Proposition 2.

One challenge in estimating valuations when the impression constraint binds is that  $D$ , the distribution of maximum of competing bids, is a function of others' bids. As such, advertisers need to form beliefs about the bids of other advertisers. In estimation, this maximum is observed and equivalent to the rational expectation of the advertisers' regarding the maximum of the competitors' bids. The estimation procedure is outlined in the online Appendix C.1.

### 4.3 Institutional Details

This sub-section discusses four institutional aspects of the data that warrant additional attention: (i) metrics (e.g., payments, number of impressions won) are only available as daily averages instead of at the auction (impression) level, (ii) the number of potential bidders,  $n$ , are not directly observed, (iii) how the minimum impression level and campaign lengths are

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<sup>35</sup> The observed characteristics affecting the valuation are discrete in our context, such as site, ad type (ad location, ad size, device), and time (month). Although the dimension of  $Z$  considered is large, we also have many observations per given particular combination of  $Z$ , enabling non-parametric estimation. If the covariates are continuous, semi-parametric approaches such as single-index models (e.g., the density-weighted derivative estimator in Powell et al. 1989, the maximum rank-correlation estimator in Han 1987) can instead be used to reduce the curse of dimensionality.

operationalized, and (iv) which data points are included in  $Z$ .

**Daily Aggregate Data** The metrics (e.g., payments, # impressions won) provided by the ad exchange are aggregated to day level, and are not available to this or other publishers at more granular levels. More specifically, an observational unit in the exchange data represents total payments, number of impressions won, and number of clicks attained for each given advertiser-DSP-day-site-ad type. For each observational unit, the average (daily) CPM paid is calculated as (total payments / number of impressions won). This average (daily) CPM paid is used as  $d_t$  in Equation (3) in forming the estimator for  $\hat{F}_D(v)$ .

As each observation represents a different number of impressions won, we weigh the average CPM paid by the number of impressions won when estimating the distribution of valuations. That is, a data point ( $y$  average CPM paid,  $x$  number of impressions won) is treated as if there are  $x$  number of observations with  $y$  CPM payment.<sup>36</sup>

**Number of Potential Bidders** The identification strategy discussed above requires that the number of potential bidders  $n$  is known. The number of potential bidders for a given observational unit (advertiser-DSP-day-site-ad type) will be

$$n = n_1(\# \text{ advertisers with CPM payment, i.e., positive } \# \text{ impressions won}) \\ + n_2(\# \text{ advertisers with bids submitted, but zero impressions won})$$

$n_1$  is observed in the data, while  $n_2$  is observed only for the opt-in advertisers. Thus, to operationalize  $n$ , we use the the following proxy:<sup>37</sup>

$$\hat{n} = n_1(\# \text{ advertisers with CPM payment, i.e., positive } \# \text{ impressions won}) \\ + n_{2,opt-in}(\# \text{ opt-in advertisers with bids submitted, but zero impressions won})$$

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<sup>36</sup> The magnitude of the potential bias in using the aggregate data (as opposed to the impression level data) is to be examined. Our conjecture is that the variance of the valuation distribution will be biased downward when using the aggregate data, but its impact on the optimal reserve price level will be small (or minimal). We will provide a formal analysis at a later time.

<sup>37</sup>  $n_{2,opt-out}$  will be small for two reasons. First, 82% of advertisers opted-in to share their bidding information (default setting). Second, the opt-out advertisers (18%) constitute about 84% of the total revenues, meaning opt-out advertisers are highly likely to be included in  $n_1$ , which is observed.

**Minimum Impression Level and Campaign Length** When advertisers do not face binding minimum impression constraints, advertisers bid truthfully and the distribution of valuations can be recovered without the knowledge of the minimum impression level or the campaign length. In the case the minimum impression constraints bind for some advertisers, the identification strategy discussed in sub-section 4.1.2 requires more information from the researcher. More specifically, in forming the estimator in Equation (10) in online Appendix,  $\tau_\theta = \left(\frac{y_\theta}{\eta s_\theta}\right)$  term is required as an input. This term reflects the minimum winning rate the advertiser aims to attain for a given impression for a campaign. Thus,  $\tau_\theta$  is advertiser-campaign specific. In the estimation (and the counterfactual), we allow  $\tau$  to vary across advertisers, but hold constant the minimum winning rate within an advertiser across campaigns. Denoting  $k$  to be the advertiser,  $\tau_k$  is proxied by

$$\hat{\tau}_k = \min(I_{k1}, \dots, I_{km}, \dots, I_{kM}), I_{km} > 0 \forall m$$

$$I_{km} = \frac{1}{\sum_{day \in m} \mathbf{1}[i_{k,day} > 0] u_{day}} \sum_{day \in m} i_{k,day}$$

where  $i_{k,day}$  represents the impressions won by advertiser  $k$  and  $u_{day}$  represents the impressions available for sale on a given day.  $I_{km}$  in the second line constructs the winning rate for a given month  $m$ , conditional on participating in the exchange. In other words,  $\hat{\tau}_k$  is computed as the minimum of the monthly winning rates observed in the data, conditioned on the advertiser participating in the exchange. Accordingly, the optimal bidding strategy profile  $\boldsymbol{\mu}|\mathbf{Z}$ , will be recovered at the advertiser level, instead of advertiser-campaign level.

The solution to the publisher's optimal reserve price in Equation (2) also involves the term  $\delta_\theta = p_\theta s_\theta$  which weights the fraction of each type winning the auction (recall,  $\delta_\theta$  is the probability mass of the bidding strategy profile  $\boldsymbol{\mu}_\theta$ , where this mass is determined as the product of the type probability,  $p_\theta$ , and the campaign length for this type,  $s_\theta$ ). We do not observe  $p_\theta$  and  $s_\theta$ , but can nonetheless compute  $\delta_\theta$  at the advertiser-type level (that is, setting  $\theta = k$ ). We do this by approximating the weight for each advertiser by its observed

share of participation. Thus, we use below proxy for  $\delta_{\theta=k}$

$$\hat{\delta}_k|Z = \frac{\sum_{day} \mathbf{1}[i_{k,day} > 0]}{\sum_k \sum_{day} \mathbf{1}[i_{k,day} > 0]}$$

where  $i_{k,day}$  represents the impressions won by advertiser  $k$  on a given day.  $\mathbf{1}[i_{k,day} > 0]$  takes value 1 if the advertiser  $k$  wins a positive amount of impressions on a given day. We include ‘month’ as the observable characteristics in  $Z$ , so the summation is done over days within a month. This  $\hat{\delta}_k|Z$  is the weight advertiser  $k$  (with the bidding strategy profile  $\mu_k$ ) plays toward the platform’s revenue given  $Z$ .

**Observed Covariates** The distribution of valuations is estimated separately for each combination of  $Z$ s to control for heterogeneity. The observed covariates considered in  $Z$  are

- Site: 20 U.S. based sites considered in our analysis
- Ad location: above-the-fold (ATF), MID, below-the-fold (BTF), no info
- Ad size: 300x250, (728x90, 970x66), 320x50, 300x600. These are the sizes conforming to Interactive Advertising Bureau (IAB) standard guideline and are most commonly used by the advertisers.<sup>38</sup>
- Device: desktop, mobile, tablet
- Month: controls for seasonality

In sum, we estimate the distribution of valuations for each 11,520 (20x4x4x3x12) combination of  $Z$ s.<sup>39</sup>

## 5 Results

This section outlines the advertiser bidding model results used to infer the advertiser valuation distribution. Recall, estimation recovers i) the distribution of the underlying advertiser valuation  $F_V$ , and ii) the vector of optimal Lagrangian multipliers  $\boldsymbol{\mu}^* = (\mu_1^*, \dots, \mu_\theta^*, \dots, \mu_\Theta^*)$

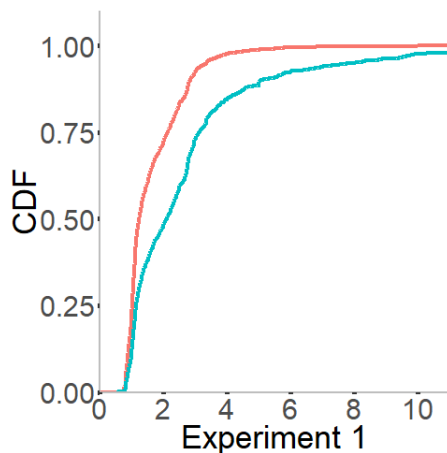
<sup>38</sup> <https://www.iab.com/guidelines/>

<sup>39</sup> Advertiser (or DSP)-specific distribution of valuations will be explored in the future to control for the heterogeneity across advertisers (or DSPs).

which reflects the tightness of the advertisers’ minimum impression constraints. The pair  $(F_V, \mu^*)$  is obtained for each combination of observables  $\mathbf{Z}$  (discrete space of site-ad location-ad size-device-month). Below we report the estimates for the treatment groups in the experimental data in the month prior to the experiment (09/2017).

### 5.1 The Valuation Distribution, $F_V$

Figure 5: Advertiser Valuations Distribution: Experiment 1



Note: The blue line is the recovered advertiser valuations distribution ( $F_V$ ) and the red line is the observed payments ( $F_D$ ).

The cumulative density function of the advertiser valuations,  $F_v$ , is plotted in Figure 5 for Experiment 1. The cdf of the advertiser valuations ( $F_V$ , blue line) is recovered from the observed payments ( $F_D$ , red line).<sup>40</sup> This figure shows how valuations exceed payments. For example, about 40% of advertisers have a valuation of less than \$2.00, and 60% of advertisers pay less than \$2.00.

In Table 6, the first and the second columns respectively represent the mean and the standard deviation of the advertiser valuations for each treatment group in the experiment. The standard deviation of the means for the distributions is large (2.84). In other words, the valuation distributions appear to vary by observables across cells such as (site-ad location-ad size-device-month), implying that different reserve prices should be set for different auctions.

<sup>40</sup> The recovered cdf of the advertiser valuations for all experiments are included in Figure 8 in online Appendix.

Table 6: Advertiser Valuations  $F_V$ : Mean and Standard Deviation

Experiment	Mean	SD
1	3.08	5.72
2	2.58	3.84
3	2.96	3.69
4	4.07	4.84
5	2.71	3.27
6	2.74	3.79
7	3.19	3.48
8	15.0	6.76
9	4.77	5.26
10	8.11	4.25
11	2.48	2.98
12	3.04	4.28
(Weighted) Mean	3.94	4.17
(Weighted) SD	2.84	0.98

Note: The weight used is the number of impressions won for each experiment.

## 5.2 The Impression Constraint, $\mu^*$

Recall,  $\mu^*$  is the premium advertisers pay to meet the minimum impression constraint (over the solution where there exists no such constraint). The first column in Table 7 reports the percentage of advertisers bound by the minimum impression constraint (i.e., those with positive Lagrangian multipliers  $\mu_k^* > 0$ ) at  $r = 0$ . On average, about 19% of the advertisers face binding minimum impression constraints during the prior experimental period when  $r = 0$ . Moreover, as the average advertiser valuation for an impression is about \$3.94, a “back of the envelope” calculation suggests that advertisers bid around 20% ( $0.78/3.94$ ) higher than their true valuations because of the binding minimum impression constraint when there is no reserve,  $r = 0$ . Also of note, the standard deviations of  $\mu$  are high (see fifth column in Table 7), meaning the cost of the constraint varies significantly across advertisers.

The percent of advertisers bound by the constraint increases from 19 to 32 as the publisher increases the reserve prices. As a result, the cost of the constraint increases from \$0.78 to \$0.99 and the overall cost of the constraint across advertisers grows substantially, from 20% to 25%.



Table 7: The Impression Constraint,  $\mu^*$

Experiment	% Advertisers Constrained		$\mu$ : Mean		$\mu$ : SD	
	$r = 0$	$r^*$	$r = 0$	$r^*$	$r = 0$	$r^*$
1	19.0	33.5	0.66	0.77	4.68	3.33
2	15.3	36.1	0.54	0.96	2.54	3.51
3	17.1	35.8	0.62	1.08	2.27	3.88
4	17.4	22.0	0.93	0.97	3.45	3.42
5	27.8	38.4	0.72	0.78	15.3	15.3
6	26.1	56.4	0.43	0.52	5.53	5.39
7	19.4	26.5	0.88	0.90	5.51	3.85
8	9.19	9.2	1.26	1.35	5.87	5.80
9	21.4	45.9	1.34	1.85	5.63	5.59
10	18.2	17.7	1.27	1.43	3.06	3.92
11	16.7	37.5	0.62	1.08	2.70	3.55
12	26.1	55.1	0.79	0.91	11.1	10.9
(Weighted) Mean	18.7	31.7	0.78	0.99	4.94	5.07
(Weighted) SD	4.35	10.7	0.24	0.22	3.91	3.66

## 6 Setting Reserve Prices

In this section, we compute the optimal reserve price to maximize the publisher’s revenues in the ad exchange auctions. Based on the recovered advertiser distribution  $F_V$ , the optimal reserve can be obtained by solving the publisher’s optimization problem prescribed in Equation (2).

### 6.1 Solving for the Optimal Reserve Price

This sub-section discusses the numerical approach used to solve for the optimal reserve price. For each experimental group, we compute two reserve prices. The first reserve price,  $r_{nc}^*$  is calculated under the assumption that advertisers bid truthfully (i.e.,  $\mu^* = 0$ ). This can be simply done by solving the implicit function in Equation (5) in online Appendix.

The second reserve price,  $r^*$ , is calculated with the minimum impression constraint in place by solving the publisher’s optimization problem in Equation (2). The second reserve price  $r^*$  takes into account advertisers’ best responses with the minimum impression constraint in FMFE.

The computation of the second reserve price requires updating advertisers’ beliefs on  $D$  to ensure that advertisers’ beliefs are consistent with the bidding profile  $\mu$  at the new reserve

price  $r$  considered. Thus solving the optimal reserve price  $r^*$  with the minimum impression constraint involves embedding the iterative best-response algorithm to find FMFE under the new reserve price. The detailed procedure is included in online Appendix C.2.

## 6.2 Optimal Reserves

Table 8: Policy Simulation Results

Experiment	Reserve Price Bias $\left(\frac{r_{n.c.}^* - r^*}{r^*}\right)$	Profit Loss
1	-18.2	4.55
2	-59.3	29.3
3	-48.1	12.7
4	-28.6	5.90
5	-33.2	14.3
6	-5.71	5.50
7	-7.41	1.04
8	0.0	0.0
9	-40.0	17.7
10	-5.56	0.09
11	0.0	0.0
12	-4.98	4.98
(Weighted) Average	-27.2	9.09

Table 8 details the predicted reserve price bias and the profit loss arising from ignoring the minimum impression constraint across all the experimental cells. The reserve price bias is calculated as the (optimal reserve without the constraint - optimal reserve with the constraint)  $\div$  (optimal reserve with the constraint). The profit loss is calculated as the difference in profits when setting the optimal reserve with the constraint and without. On average, the optimal reserve price calculated with no constraint is about 27% lower than the optimal reserve calculated with the minimum impression constraint, but can be as high as 59%. Further, the profit loss in ignoring this constraint is calculated to be around 9% across experiments, but can be as high as 29%. From this, we conclude that the minimum impression constraint can lead to even more substantial gains in publisher revenues than a naive approach that assumes no constraint.

## 7 Conclusion

With the rapid growth in display advertising markets, there is an increasing value in characterizing the advertisers’ valuations for ad impressions, and how advertiser strategies are affected by practical constraints (such as reach or budget) or by the reserve price in advertising exchange markets. Taking the perspective of the publisher, we consider how reserve prices should be set for auctions when selling display advertising impressions through these ad exchanges.

The optimal reserve is a function of advertiser valuations, and auction theory has established that bidding one’s true valuation is the weakly dominant strategy in static, second-price, sealed-bid auction with standard assumptions. Though the one-shot game setting is tractable and often adopted in models of advertiser bidding (e.g., Sayedi 2017, Celis et al. 2014, Johnson 2013), our discussions with industry participants indicate that advertisers recognize that they buy multiple impressions across multiple auctions with some constraints on reach or budget. This implies that advertisers view ad buying as playing repeated games with some constraints, which would then lead their bids to deviate from truth-telling.

In a series of experiments, setting the reserve price under the assumption advertisers play a one-shot game without any constraints is shown to increase publisher’s revenue substantially, by 32% (notably, at no additional cost to the publisher). By manipulating the reserve in this fashion, we test the extent to which reach or budget constraints (across multiple auctions) affect advertisers’ bidding behaviors. Experimental findings indicate that increasing the reserve price increases advertisers’ bid CPMs and total payments, while it does not have an impact on the total number of impressions won by the advertisers. We show these patterns are most consistent with advertisers playing repeated auctions with minimum impression constraint.

Subsequently, we construct an advertiser bidding model that incorporates the minimum impression constraint. The model builds on the notion of a fluid mean-field equilibrium developed in Balseiro et al. 2015, which well approximates the rational behavior of thousands of

advertisers competing in repeated auctions with some constraints. We extend this theoretical framework to incorporate the minimum impression constraint, and suggest estimation and identification strategies in applying it to our empirical context. Our counterfactual results show that the reserve price solved without imposing the minimum impression constraint is 27% lower than the optimal reserve level with the minimum impression constraint. Because ignoring the minimum impression constraint biases the solution downward, the profit loss in ignoring this constraint is found to be about 9%. We plan to conduct an additional set of experiments to validate this pricing policy

While this paper addresses a question of growing economic importance with a novel dataset, a number of additional extensions are possible. In particular, publishers often sell advertising inventory via direct sales as well as RTB. Direct selling involves advance sale of a bundle of impressions directly to the advertiser at a fixed price. Extending our research to consider optimal joint pricing (e.g., fixed price in the direct, reserve price in the ad exchange), and whether inventory should first be made available to one channel or another, are interesting directions for future research, toward which we are currently working.<sup>41</sup> A second question of interest is motivated by noting that advertiser valuations are incumbent upon the information available to the advertisers about the impression. This raises the question of whether and how much information a publisher should share with an exchange.

To the best of our knowledge, this paper is among the first to empirically consider the issues of pricing in display advertising markets. Given our initial results and the growth in these markets, we hope this and future research will continue to yield economically meaningful implications for publishers in these rapidly growing markets and lead to more research in this area.

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<sup>41</sup> We consider reserve prices, taking direct sales decisions made as given (e.g., price, number of impressions bought and sold in direct). Taking the direct channel as given is an assumption that mirrors the structure of the market we consider, where the impressions are sold in the ad exchange after they do not sell in the direct sales channel. Thus, our solution to the optimal reserve prices can be viewed as the sub-game perfect equilibrium solution to the second-stage, taking the first-stage decisions as given.

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# Online Appendix

## A Field Experiment

### A.1 Experiment Design

A series of experiments were designed to manipulate the reserve prices in the exchange channel. Data were collected for the period 08/01/2017 - 12/03/2017, where 08/01/2017 - 10/15/2017 constitute the ‘Prior’ period and 10/19/2017 - 12/03/2017 constitute the ‘Post’ period where the changes in reserve prices took place for the treatment group. The experiments were conducted on two chosen websites that were closest in terms of contents, user demographics, revenues, and number of impressions (user visits).

#### A.1.1 Treatment and Control Groups

The experiments were designed to run across different ad types including (i) site (site1 and site2), (ii) device (desktop, mobile, tablet, no info), (iii) ad location (ATF, MID, BTF, no info, front door), and (iv) size (300x250, 728x90, 970x66, 320x50, 300x600, 970x250). Twelve experiments were constructed; each constituted a pair, closest in ad characteristics, that was randomized either into the treatment or the control group. For example, (site1, desktop, BTF, 300x250) and (site1, desktop, ATF, 300x250) were paired for the first experiment, and the randomly chosen (site1, desktop, BTF, 300x250) was assigned to the treatment group, whereas (site1, desktop, ATF, 300x250) was assigned to the control group. Table 9 shows the full list of the experiments and the corresponding treatment and control group characteristics.

Table 9: Treatment and Control Groups

Id	Device	Site and Position		Inventory Size	
		Treatment	Control	Treatment	Control
1	Desktop	Site1 BTF	Site1 ATF	300x250	300x250
2	Desktop	Site2 MID	Site2 ATF, BTF	300x250	300x250
3	Mobile	Site1 No Info	Site2 MID, BTF	300x250	300x250
4	Mobile	Site2 ATF	Site2 MID, BTF	300x250	300x250
5	Desktop	Site1 ATF	Site1 BTF	728x90, 970x66	728x90, 970x66
6	Tablet	Site1 ATF, BTF	Site1 No Info	728x90, 970x66	728x90, 970x66
7	Mobile	Site2 MID, BTF	Site1 No Info, Site2 ATF	320x50	320x50
8	Desktop	Site1 ATF	Site2 ATF	300x600	300x600
9	All	Site1 Front Door BTF	Site1 Front Door BTF	300x250	728x90
10	Desktop	Site1 ATF	Site1 BTF	970x250	970x250
11	Tablet	Site1 No Info	Site2 ATF, MID, BTF	300x250	300x250
12	Desktop	Site2 BTF	Site2 ATF	728x90, 970x66	728x90, 970x66



### A.1.2 Optimal Reserve Price

For purposes of the experiment, advertiser valuations are estimated assuming advertisers use truth-telling strategies, and the optimal reserve prices are calculated conditioned on this assumption.

Under the standard second-price, sealed-bid auction, where advertisers bid truthfully, their valuation distribution can be identified (sub-section 4.1.1) and estimated (sub-section 4.2.1) from the observed payment data for each given ad characteristics.

The publisher can maximize the revenue from the exchange channel by choosing the reserve price optimally. The optimal reserve price  $r_{nc}^*$  can be expressed as

$$r_{nc}^* = c + \frac{[1 - F_V(r_{nc}^*)]}{f_V(r_{nc}^*)} \quad (5)$$

where  $c$  is publisher’s valuation (Riley and Samuelson 1981).  $F_V$  and  $f_V$  are cdf and pdf of advertiser valuation distribution. For each of the twelve experiments, we use the estimated distribution  $\hat{F}_{V|Z}$  and  $\hat{f}_{V|Z}$  to calculate  $r_{nc}^*(Z)$ .

### A.1.3 Pre-Trend

The DiD analysis requires common trends. Below we plot trends for CPM, where the vertical dotted line indicates the start date of the experiment. Although control and treatment groups have different mean levels for some experiments, in general the trends are parallel. In many cases, the CPMs for both the treatment and control groups increase toward the end of the year due to seasonality.

## A.2 Supplementary Field Experiment Analyses

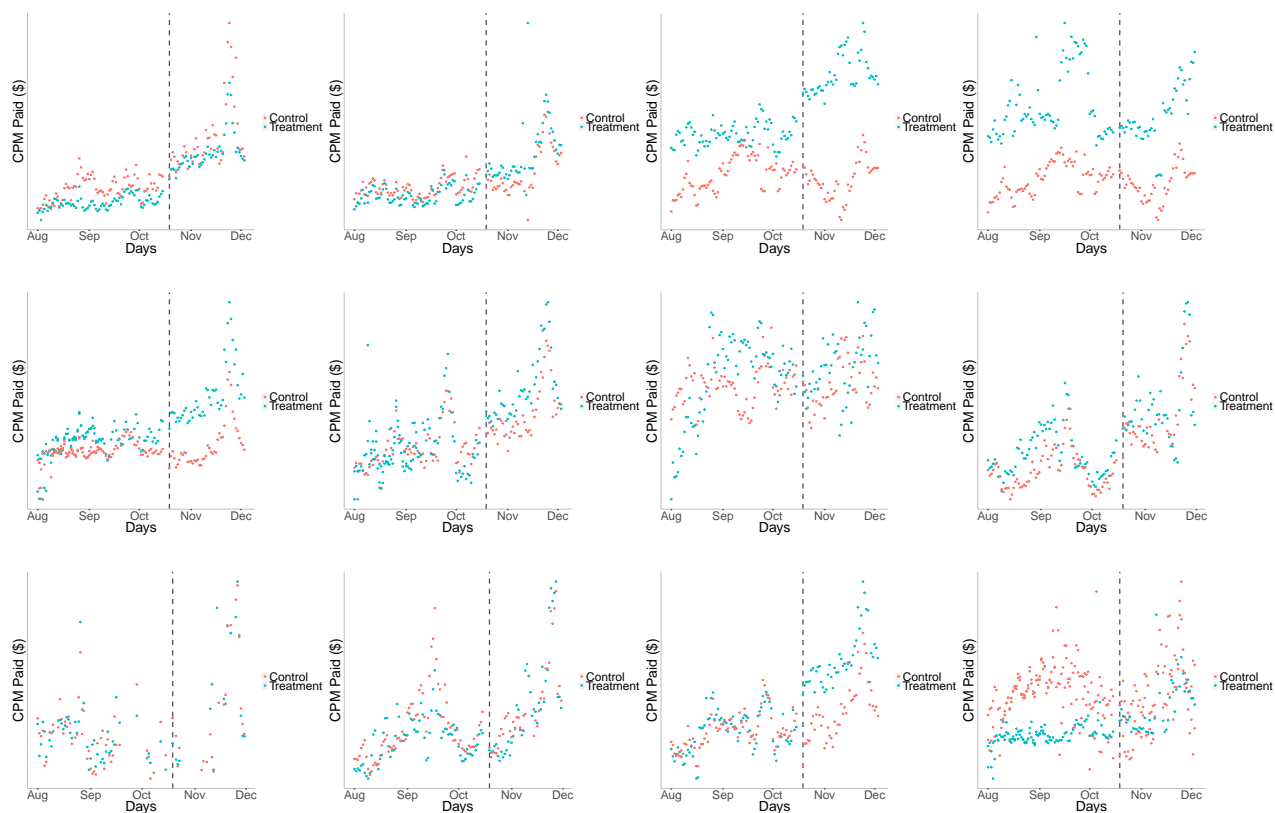
### A.2.1 Experimental Results on eCPM

Table 10: Treatment Effect on eCPM

DV= eCPM	Model 1		Model 2		Model 3		Model 4	
	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
Treated × Post	0.14*	0.03	0.12*	0.02	0.11*	0.02	0.12*	0.01
Treated	0.07*	0.01	0.09*	0.01	0.11*	0.01	–	
Treated × Experiment	–		–		–		yes	
Post	–		–		–		–	
Day fixed effect	yes		yes		yes		yes	
Experiment fixed effect	–		yes		yes		yes	
Site-Adtype controls	–		–		yes		yes	
Adjusted $R^2$	0.04		0.26		0.64		0.82	
$N = 8,763, \quad *p < 0.01$								

Next, the results from a formal difference-in-difference (DiD) analysis on eCPM are reported, in which observables are controlled for. The outcome measure considered is *eCPM*

Figure 6: Pre-Post Trends



(multiplied by a common, multiplicative constant for confidentiality) which is estimated to be positive and significant.<sup>42</sup> Based on the DiD estimate, the increase in revenue is

$$\text{Increase in revenue} = \frac{0.11}{0.38} \simeq 30\%$$

where 0.38 is the baseline eCPM (again scaled for confidentiality) for the treatment group in the ‘Prior’ period.

### A.2.2 Robustness Checks

The first column in Table 5 reports that bid CPMs are higher when the reserve prices increase. One concern in using bid CPM as the dependent variable is that this measure is only available for the opt-in advertisers who share their data with the publishers. Therefore to account for this selection issue, we also consider the payment CPM, which is the second order statistics of the bid distribution under the second-price auctions. The second column in Table 11 shows that the payment CPM also increases with respect to the increase in the reserve prices.

<sup>42</sup> From the experiment, we have 7,054,582 observations at the advertiser-DSP-day-site-ad type level. Aggregation is necessary for this DiD analysis, because the denominator in eCPM (i.e., ‘impressions supplied to exchange’) can only be defined at day-site-ad type level. Thus, the data are aggregated across advertisers and DSPs, leaving us 8,763 observations. Each observation is weighted by the number of ‘impressions supplied to exchange’ in running the weighted least square regression.

Table 11: DiD Regression Results

DV	Bid CPM	Payment CPM	Bid CPM ( $> r$ )	Payment CPM ( $> r$ )	CPM ( $> r$ )	# Impressions Won	Total Payment
Treated $\times$ Post	0.06** (0.01) Y	0.31** (0.01) Y	0.10* (0.05) Y	0.17** (0.01) Y	-0.50 (0.89) Y	0.007** 0.003 Y	
Treated $\times$ Experiment	Y	Y	Y	Y	Y	Y	
Day fixed effect	Y	Y	Y	Y	Y	Y	
Experiment fixed effect	Y	Y	Y	Y	Y	Y	
Site-Adtype controls	Y	Y	Y	Y	Y	Y	
Advertiser fixed effect (Top 1000)	Y	Y	-	Y	Y	Y	
Advertiser fixed effect (All)	-	-	Y	-	-	-	
DSP fixed effect	Y	Y	Y	Y	Y	Y	
Adjusted $R^2$	0.47	0.60	0.63	0.60	0.35	0.33	
$N$	4,712,375	7,054,582	35,690	3,062,841	7,054,582	7,054,582	

Note: \* $p < 0.1$  \*\* $p < 0.05$

If the experimental reserves exceed an advertiser’s valuation, the advertiser might not submit a bid in the treatment condition. In this instance, full bids are observed for the control group, but only the truncated bids (exceeding the reserve price) are observed for the treatment group in the ‘Post’ period. To account for this from of truncation, both the control and treatment group (and both the pre- and post-) data are truncated at the reserve price in the DiD analysis. If advertisers bid their true valuations, bids in this truncated region will stay the same, pre- and post- the experiments. However, in the third column in Table 11, bid CPMs increase with the reserve prices.

Similarly, the payment CPM for the treatment group in the ‘Post’ period will, by definition, be truncated at the reserve price set by the publisher. To account for this effect, we also run DiD analysis for the payment data conditioned on being above the reserve price (i.e., payments are left-truncated at the post period, treatment group reserve level, for both the control and treatment group and for both the pre- and post-periods). Results are robust inasmuch as the second-highest bids increase with the reserve prices, conditioned on payments exceeding the reserve price set for the experiment (the fourth column).

In sum, the result that advertisers bid higher when the publisher increases the reserve price is robust to various controls, measures, and assumptions.

## B Model

### B.1 Proofs

#### B.1.1 Proposition 1

The proof for Proposition 1 follows the steps established in Balseiro et al. 2015 Section A.1. First, the dual of the primal problem in Equation (1) is introduced using a Lagrange multiplier for the minimum impression constraint. Second, the first-order conditions are derived to determine the solution for the dual problem.

**Step 1:** The Lagrangian for type  $\theta$  is denoted as

$$\begin{aligned} \mathcal{L}_\theta(b, \mu) &= \eta s_\theta E_{V,D} [\mathbf{1}\{b(V) \geq D\} (V - D)] + \mu [\eta s_\theta E_{V,D} [\mathbf{1}\{b(V) \geq D\}] - y_\theta] \\ &= \eta s_\theta E_{V,D} [\mathbf{1}\{b(V) \geq D\} (V - D + \mu)] - \mu y_\theta \end{aligned} \quad (6)$$

where a Lagrange multiplier for the minimum impression constraint is  $\mu \geq 0$ . The dual problem (converting from maximizing the advertiser’s objective function given its minimum impression constraint in Equation (1) to minimizing the Lagrangian multipliers while maximizing the

objective function) is given by

$$\begin{aligned}
\Psi_\theta(\mu) &= \inf_{\mu \geq 0} \sup_{b(\cdot)} \mathcal{L}_\theta(b, \mu) \\
&= \inf_{\mu \geq 0} \left\{ \eta s_\theta \sup_{b(\cdot)} \left\{ E_{V,D} [\mathbf{1} \{b(V) \geq D\} (V + \mu - D)] \right\} - \mu y_\theta \right\} \\
&= \inf_{\mu \geq 0} \left\{ \eta s_\theta E_{V,D} [\mathbf{1} \{V \geq D - \mu\} (V - (D - \mu))] - \mu y_\theta \right\}
\end{aligned} \tag{7}$$

the inf is the Lagrangian minimization step and the sup is the goal maximization step. The equality in the last line comes from the fact that the inner optimization problem is similar to an advertiser's problem who faces value  $v + \mu$  and seeks to maximize its expected utility in the second-price auction so that bidding truthfully becomes optimal (consider Equation 1 without the constraints). That is for any given multiplier  $\mu \geq 0$ , the inner expectation term is maximized with the policy  $b(V) = V + \mu$ . Further, the term within the expectation in the last line is convex in  $\mu$ , and the expectation preserves convexity, leading to a convex dual problem.

**Step 2:** The first order condition of  $\Psi_\theta(\mu)$  with respect to  $\mu$  (that is, the FOC for the  $\inf_{\mu \geq 0} \{\cdot\}$ ) is given by

$$(d/d\mu)\Psi_\theta(\mu) = \eta s_\theta E_{V,D} [\mathbf{1} \{V \geq D - \mu\}] - y_\theta = 0 \tag{8}$$

To explain the solution to this FOC, we begin by noting that  $\Psi_\theta(\mu)$  is convex in  $\mu$ . If the constraint does not bind (i.e.,  $\eta s_\theta E_{V,D} [\mathbf{1} \{V \geq D\}] \geq y_\theta$ ), then  $(d/d\mu)\Psi_\theta \geq 0$  at  $\mu = 0$ . This condition implies the function  $\Psi_\theta(\mu)$  is increasing in  $\mu$  for all  $\mu \geq 0$ , such that the function is minimized at  $\mu^* = 0$  (has a corner solution). Intuitively, the Lagrangian multiplier can be interpreted as the cost of the impressions constraint; If the constraint does not bind, the constraint is costless.

On the other hand, when the constraint binds (i.e., the optimal unconstrained number of impressions is less than the minimum impression level, that is  $\eta s_\theta E_{V,D} [\mathbf{1} \{V \geq D\}] < y_\theta$ ), then  $(d/d\mu)\Psi_\theta(\mu)$  takes a negative value at  $\mu = 0$ . As  $\mu \rightarrow \infty$ ,  $(d/d\mu)\Psi_\theta(\mu)$  converges to a positive value  $\eta s_\theta - y_\theta > 0$ .<sup>43</sup> Thus, there exists a unique interior solution  $\mu^* > 0$  for  $(d/d\mu)\Psi_\theta(\mu^*) = 0$  as  $\Psi_\theta$  is a convex function in  $\mu$ .

Moreover, the complementary slackness conditions hold with the bidding function  $b^*(V) = \beta_\theta^F = v + \mu^*$  and the optimal multiplier  $\mu^*$  such that:

$$\mu^* \left[ \eta s_\theta E_{V,D} [\mathbf{1} \{\beta_\theta^F(V) \geq D\}] - y_\theta \right] = 0,$$

That is, either i) the minimum impression constraint binds or ii) the Lagrangian multiplier  $\mu^*$  is 0.

Lastly, there is no duality gap. That is, the solution to the dual problem in Equation (7)

<sup>43</sup>  $(\eta s_\theta - y_\theta)$  is a finite positive number as we constrain  $\eta s_\theta > y_\theta$  when defining the optimization problem in sub-section 3.3 (i.e., the minimum impression level is lower than the total available impressions).

is optimal for the original problem in Equation (1).

$$\begin{aligned}
& \eta s_\theta E_{V,D} [\mathbf{1} \{b(V) \geq D\} (V - D)] \\
&= \mathcal{L}_\theta(\beta_\theta^F, \mu^*) - \mu^* [\eta s_\theta E_{V,D} [\mathbf{1} \{\beta_\theta^F(V) \geq D\}] - y_\theta] \\
&= \mathcal{L}_\theta(\beta_\theta^F, \mu^*) \\
&= \Psi_\theta(\mu^*)
\end{aligned}$$

where the second equality follows from the complementary slackness conditions, and the last equality follows from the fact that  $\Psi_\theta(\mu^*) = \sup_{b(\cdot)} \mathcal{L}_\theta(b, \mu^*)$  and the optimal bid function  $\beta_\theta^F$  solves the latter problem.

### B.1.2 Proposition 2

Once we establish the optimal bidding function and the optimal multiplier as above, Proposition 2 follows from Proposition 4.1 in Balseiro et al. 2015, in which they characterize the equilibrium. Since we analyze the minimum impression constraint without the participation constraint, the existence and the characterization of the equilibrium are valid only when the participation constraints do not bind in equilibrium. Readers are referred to their proof in the supplemental material ([http://dx .doi.org/10.1287/mnsc.2014.2022](http://dx.doi.org/10.1287/mnsc.2014.2022)).

## B.2 Theoretical Predictions

In this appendix, we outline the intuition for the predictions in Table 3 in Section 2. Recall, these predictions indicate how advertiser behaviors change as the reserves increase from 0 to a reserve level  $r_{nc}^*$ , that is the optimal reserve price under the assumption that advertisers do not face binding impression constraints.

In the subsequent analysis, we consider several cases: i) when the constraint binds neither at 0 nor at  $r_{nc}^*$  (not bind, not bind), ii) when the constraint does not bind at 0 but does bind at  $r_{nc}^*$  (not bind, bind), and iii) when the constraint binds at both the 0 and  $r_{nc}^*$  reserve levels (bind, bind). We consider two types of constraints: a maximum budget constraint and a minimum impression constraint. The minimum impression constraint and the maximum budget constraint are each considered in isolation. That is when we consider the budget constraint, we assume that the minimum impression constraint does not bind in both ( $r = 0$ ) and ( $r_{nc}^* > 0$ ).

### B.2.1 No Binding Impression or Budget Constraint (Not Bind, Not Bind)

When the underlying state is (not bind, not bind), the advertiser bidding model collapses to the standard second-price auction without the constraint. In this case, advertisers bid their true valuations, and the distribution of bids will be invariant regardless of the reserve price level. The probability of winning an impression decreases at higher  $r_{nc}^*$ , but the total

payment increases as  $r_{nc}^*$  maximizes publisher's revenues.

### B.2.2 When Impression Constraints Bind ((Not Bind, Bind) or (Bind, Bind))

#### Impression Constraints: The Effect of the Reserve Price on the Optimal Bid

The equilibrium Lagrangian multiplier  $\mu^*$  increases monotonically with the increase in  $r$  (until the participation constraint binds). When the minimum impression constraint binds, the optimal  $\mu^* > 0$  satisfies  $y_\theta - \eta s_\theta E_{V,D} [\mathbf{1} \{V + \mu^* \geq D\}] = 0$  (see Equation 8). An increase in the reserve price will increase  $D$  (the steady-state maximum of the competitors' bids), and to offset this effect,  $\mu^*$  needs to increase as well to satisfy the equality constraint. As the optimal bidding strategy is derived as  $\beta_\theta^F = v + \mu^*$ , an increase in  $r$  increases  $\mu^*$ , which in turn increases bids.

#### Impression Constraints: The Effect of the Reserve Price on Number of Impressions

**Won and the Total Payment** When at least some advertisers face the case of (not binding, binding), the number of impressions won by advertisers will decrease as the reserve prices increase. This is because some advertisers will face a binding impression constraint at  $r_{nc}^*$  so they cannot buy as many impressions as they used to when there was no reserve,  $r = 0$ .

The direction of the total payments across the advertisers is ambiguous when reserves increase in the (not binding, binding) condition because there are opposing effects. Although the total number of impressions won decreases, advertisers increase their bids with the increase in the reserve price leading to a higher payment CPM per impression sold.

Finally, when the advertiser faces the (binding, binding) condition, the number of impressions won do not change, as the advertiser cannot decrease the number of impressions to buy with the increase in the reserve price (as the constraint does not allow this).<sup>44</sup> Accordingly, the total payment will increase with the increase in the reserve price, with the increase in advertisers' bids.

### B.2.3 When Budget Constraints Bind ((Not Bind, Bind) or (Bind, Bind))

#### Budget Constraints: Optimal Bidding Strategy

Balseiro et al. 2015 establish that the optimal bidding strategy when advertisers face the maximum budget constraint is

$$\beta_\theta^F(v|F_D) = \frac{v}{1 + \mu^*} \quad (9)$$

where  $\mu^*$  is

$$\begin{cases} \mu^* = 0 & \text{if } b_\theta > \eta s_\theta E_{V,D} [\mathbf{1} \{V \geq D\} D] \\ \eta s_\theta E_{V,D} [\mathbf{1} \{V/(1 + \mu^*) \geq D\} D] - b_\theta = 0 & \text{if } b_\theta \leq \eta s_\theta E_{V,D} [\mathbf{1} \{V \geq D\} D] \end{cases}$$

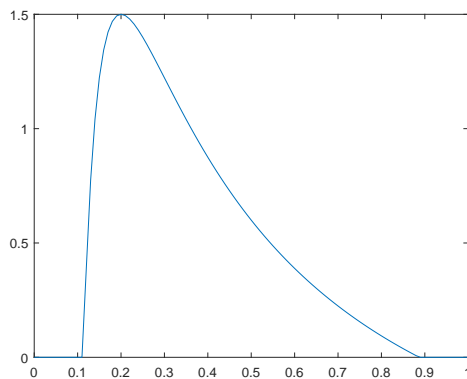
and  $b_\theta$  is the maximum (constrained) budget.

<sup>44</sup> The number of impressions can decrease if the participation constraint binds for some advertisers.

**Budget Constraints: The Effect of Reserve Price on Optimal Bid** When the advertiser faces the case of a (not bind, bind) budget constraint as the reserve price increases from 0 to  $r_{nc}^*$ , the bidding strategy will change from  $v$  to  $\frac{v}{1+\mu^*}$  where  $\mu^* > 0$ . Thus, the bid will decrease in this case.

When the advertiser faces (bind, bind) constraint as the reserve price increases from 0 to  $r_{nc}^*$ , the effect on the bid is ambiguous. This is because  $\frac{\partial \mu^*}{\partial r}$  is not monotonic. For example, Figure 7 shows the change in  $\mu^*$  (y-axis) with respect to the change in  $r$  (x-axis), when  $v \sim U[0, 1]$  and  $b = 0.1$ , when there is one advertiser. The plot shows that  $\mu^*$  first increases then decreases in the range  $[0, r_{nc}^*] = [0, 0.5]$ .

Figure 7: Optimal Bidding Strategy with a Maximum Budget Constraint



Note: The figure shows the change in  $\mu^*$  (y-axis) with respect to the change in  $r$  (x-axis).

**Budget Constraints: The Effect of Reserve Price on Number of Impressions Won and Total Payment** When the advertiser faces the case of a (not binding, binding) budget constraint as the reserve price increases from 0 to  $r_{nc}^*$ , the number of impressions won will decrease as advertisers now shade bids and the reserve price increases (see Equation 9). The total payment across advertisers increases, because advertisers spend more and the budget constraint becomes binding.

When the advertiser faces the case of a (binding, binding) budget constraint, the total payment will stay the same (at the binding budget level), but the effect on the number of impressions won is ambiguous because bid CPM may or may not increase with respect to the change in the reserve price.

#### B.2.4 Other Constraints

When advertisers set pacing options, we expect the results to be similar to the budget constraint case, as pacing plays a similar role to setting an even budget over a specified time



period. Finally, advertisers' learning of their own or others' valuations can either decrease or increase bid CPM depending on the signals received.<sup>45</sup>

## C Estimation and Policy Simulation

This section overviews the computational algorithms used in estimation and policy simulations.

### C.1 Computational Steps in Estimation

The estimation proceeds as follows:

**Stage 1** Denoting  $w = v + \mu$ ,  $\hat{F}_W(w)$  and  $\hat{F}_D(d)$  are non-parametrically estimated as described in Equations (3) and (4).

**Stage 2** Using the optimality condition in Proposition 2,  $\boldsymbol{\mu}^*$  is solved for the FMFE using the following algorithm.

1. Start with an arbitrary vector of multipliers  $\boldsymbol{\mu}$ . That is  $\mu_\theta^0 = \mu_\theta, \forall \theta \in \Theta$
2. Repeat

- (a) Using the estimates  $(\hat{F}_D(d), \hat{F}_W(w))$  obtained in the first stage,  $N_{sim}$  simulated values are drawn from the estimated distributions  $v \sim \hat{F}_V = \hat{F}_W(v + \mu_\theta^i)$  and  $d \sim \hat{F}_D(d)$  to construct

$$\hat{h}(\mu_\theta; \boldsymbol{\mu}^i) = \frac{y_\theta}{\eta s_\theta} - \frac{1}{N_{sim}} \sum [\mathbf{1}\{v_{sim} + \mu_\theta \geq d_{sim}\}] \quad (10)$$

This equation follows from the advertiser's optimal bidding strategy condition characterized in Proposition 2.

- (b)  $\mu_\theta^{i+1}$  solves

$$\arg \min_{\mu_\theta \geq 0} \hat{h}(\mu_\theta; \boldsymbol{\mu}^i), \quad \forall \theta \in \Theta$$

where minimizing  $\hat{h}(\mu_\theta; \boldsymbol{\mu}^i)$  over  $\mu_\theta \geq 0$  ensures that the conditions in Proposition 2 are satisfied in equilibrium

- (c) Compute the difference  $\Delta = \|\boldsymbol{\mu}^{i+1} - \boldsymbol{\mu}^i\|$  and update  $i = i + 1$

3. Until  $\Delta < \epsilon$

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<sup>45</sup> If advertisers have a common value component (Bajari and Hortacsu 2003, Hong and Shum 2006), bids are also expected to decrease (shade more) when setting the (secret) reserve price, as the seller with the reserve price is treated as one additional competitor.

## C.2 Computational Approach to Computing Optimal Reserves in the Policy Simulation

This sub-section discusses the numerical approach used to solve for the optimal reserve price. In order to find the new FMFE under the considered reserve price level, the advertisers' beliefs on  $D$  (which reflects the bids of competing advertisers) need to be updated to ensure that the advertisers' new beliefs are consistent with the bidding profile  $\boldsymbol{\mu}(D, r)$  at the new reserve price. Thus solving the optimal reserve price with the minimum impression constraint involves embedding the iterative best-response algorithm. That is, given the recovered  $F_v$ ,

1. Start with an arbitrary  $r^j$  for  $j = 0$  (we start with  $r^0 = r_{nc}^*$ , the optimal reserve when advertisers bid truthfully)
2. Repeat:  $r$ -step
  - (a) Start with an arbitrary vector of multipliers  $\boldsymbol{\mu}$ . That is  $\mu_\theta^0 = \mu_\theta, \forall \theta \in \Theta$
  - (b) Repeat:  $\boldsymbol{\mu}$ -step
    - i. Obtain  $F_D(\cdot | \boldsymbol{\mu}^i)$
    - ii.  $\mu_\theta^{i+1}$  solves
$$\arg \min_{\mu_\theta \geq 0} h(\mu_\theta; \boldsymbol{\mu}^i), \quad \forall \theta \in \Theta$$

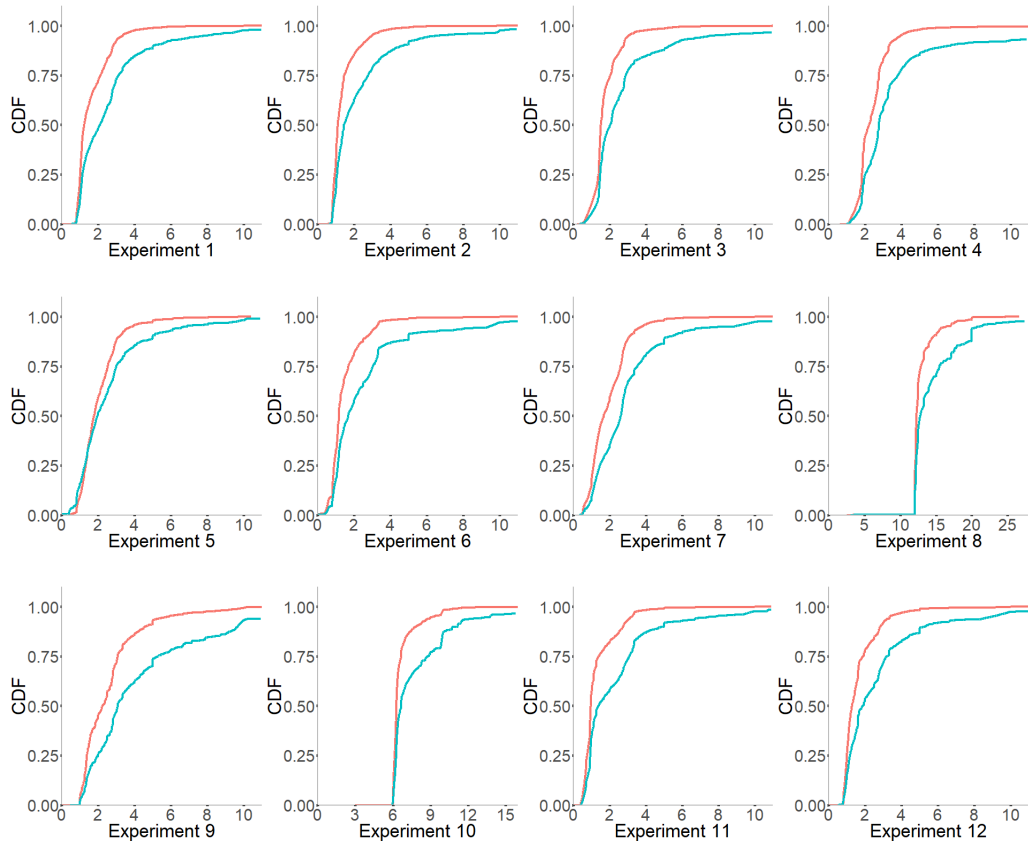
$$h(\mu_\theta; \boldsymbol{\mu}^i) = \frac{y_\theta}{\eta s_\theta} - E_{V,D}[\mathbf{1}\{V + \mu_\theta \geq D\}]$$
    - iii. Check the participation constraint  
For  $\theta$  with  $0 > E_{V,D}[\mathbf{1}\{V + \mu_\theta \geq D\}(V - D)]$ , update  $\mu_\theta^{i+1} = \bar{\mu}_\theta$  where
$$\bar{\mu}_\theta = \operatorname{argmax}_{\mu_\theta \geq 0} [0 \leq \eta s_\theta E_{V,D}[\mathbf{1}\{V + \mu_\theta \geq D\}(V - D)]]$$
    - iv. Compute the difference  $\Delta = \|\boldsymbol{\mu}^{i+1} - \boldsymbol{\mu}^i\|$  and update  $i = i + 1$
  - (c) Until  $\Delta < \epsilon$
  - (d) Optimize Equation (2) to compute the new reserve price  $r^{j+1}$
3. Until the global maximum is found

## D Results

### D.1 The Valuation Distribution, $F_V$

The cumulative density function of the advertiser valuations,  $F_v$ , is plotted in Figure 8. The cdf of advertiser valuations ( $F_V$ , blue line) is recovered from the observed payments ( $F_D$ , red line). The shape of the distributions vary substantially. In other words, the valuation

Figure 8: Advertiser Valuation Distributions



Note: The blue line is the recovered advertiser valuations distribution ( $F_V$ ) and the red line is the observed payments ( $F_D$ ).

distributions appear to vary by observables such as (site-ad location-ad size-device-month), implying that different reserve prices should be set for different auctions.